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Some Variants of Classical Multiphase Flow Problems

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I will briefly discuss three problems that have classical roots and in each case seek to add one new feature to a modern version of the problem. In the first problem the Saffman-Taylor viscous fingering problem is discussed for the case that there are geometric variations in the flow directions. We show via experiments and theory that such changes can significantly modify the stability features of the flow. In the second problem we consider the low Reynolds number motion of a hot sphere in a fluid accounting for the variations of the viscosity with temperature. We show that the Lorentz Reciprocal Theorem provides a means to construct an analytical representation of the force and torque on the sphere for the case of small viscosity variations. Finally, we present experiments of unexpected dynamics in modest Reynolds number flows at a T-junction and rationalize the results by demonstrating the connections to vortex breakdown.

1 Introduction

Fluid dynamics is a subject in which the continuum mechanical ideas apply over an enormous range of length scales: from geophysical studies of oceans, atmospheres, and planetary interiors, which occur on length scales of thousands of kilometers for Earth, to problems of swimming and flying, which occur on scales from fractions of a meter to tens of meters (e.g. birds, fish, airplanes, satellites), to medical devices, which operate at the scales of millimeters to fractions of a meter, and down to the scale of the cell (typically microns to tens of microns) and micro- and nano-fluidic devices. This range from thousands of kilometers to smaller than microns thus introduces challenges over more than 12 orders of magnitude in length scale, and the subjects include practically every science and engineering department at a university.

Single phase incompressible flows can typically be described by a Reynolds number. The presence of an interface or temperature variations introduces additional dimensionless parameters needed to characterize the problem. For example, in most low Reynolds number flows, a problem where interfacial tension matters introduces a capillary number. Our courses and textbooks highlight a variety of flow problems of this type. In this article I will discuss three problems that occur in classical contexts but identify new features when one realistic feature of the problem is changed. Some of these ideas are described in Ref. 1.

2 Variations on Classical Ideas

2.1 Viscous fingering with geometric variations in the flow direction

The viscous fingering phenomenon, generally known as the Saffman-Taylor instability, has been studied in a wide variety of systems, e.g., radial configurations, viscoelastic fluids, etc. We will highlight a role for geometry in altering the traditional stability behavior, and we've noted the similarity of these ideas to coating flow problems involving thin films on rollers[3].

In particular, we first illustrate how a droplet interacts with an obstacle in a pressure-driven two-phase flow in a microfluidic device[2]. At low speeds a droplet impacts the obstacle and eventually bypasses it through one gap without breaking. At higher speeds the droplet passes around both sides of the obstacle and breaks into two drops. We argue that the dynamics of flow through the narrow gaps on either side of the obstacle can be described as approximately one-dimensional, where an additional pressure gradient acts owing to the curvature of the meniscus, which changes its radius of curvature as it moves through a spatially varying gap[2]. We then present two-phase flow experiments in a Hele-Shaw cell with a small inclination angle and show that we can modify the Saffman-Taylor instability criterion in a flow-dependent manner that is tied to the inclination angle[3]. In this manner we can stabilize otherwise unstable viscosity variations such as when a low viscosity fluid displaces a high viscosity fluid.

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2.2 Motion of a hot particle in a viscous fluid

There are many problems of colloid science and physico-chemical hydrodynamics where temperature gradients are relevant to the motion of suspended particles. One problem of this type concerns the motion of a hot sphere in a low Reynolds number flow. The temperature changes the viscosity and hence this problem can be considered one where there is a viscosity "field", i.e. formally, a solution to this problem requires a general solution to the Stokes equations considering viscosity as a position-dependent function. Then by applying the boundary conditions for the object, we can in principle evaluate the flow field and eventually the corresponding force and torque on the object. Instead we show how to use the Lorentz Reciprocal Theorem to derive a general analytical expression for the hydrodynamic force and torque on an arbitrarily shaped translating and rotating particle for a prescribed, but otherwise arbitrary, viscosity distribution. Several examples of the motion of a sphere are outlined depending on the nature of the temperature forcing, including how a dipolar structure of a temperature distribution about a sphere leads to coupling of translation and rotation[4].

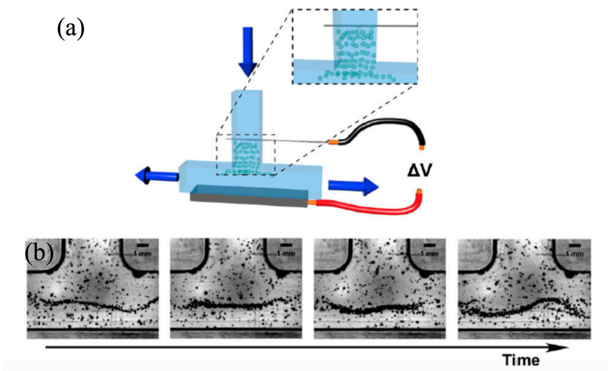

Figure 1 (a) Schematic of flow in a T-junction, with flow entering at the top and leaving both sides; air bubbles appear black. (b) A time sequence, with images taken every 125 μs, of the bubbly flow showing bubble trapping during flow of water in a T-junction; lateral size L = 4.8 mm, Re = 980. Figure reproduced from Ref. 5.

2.3 Single and multiphase flows in T-junctions and vortex breakdown

As a final example of complexity in a seemingly simple flow configuration, I describe single-phase and multiphase studies we have performed in recent years in the common engineering configuration of a T-junction and other simply branched junctions at other angles. This geometry is typical of a wide range of fluid distribution systems. To describe how we stumbled on this problem, Fig. 1 provides experimental observations of the inertially dominated flow of bubbles (in a water flow) through a T-junction when Re = 980 [5]. Naturally, one expects that the flow should transport the bubbles through the T-junction. To our surprise bubbles get trapped in the junction and accumulate. Gravity does not matter, but the density difference between the bubbles and the liquid is important.We have found no reference in the literature to this phenomenon and in fact now understand it as primarily (for a low number density of bubbles) controlled by a single-phase flow phenomenon, and bifurcation, in this channel configuration[5–7]. I will discuss the numerical and experimental evidence for the occurrence of vortex breakdown in this geometry and how such dynamics produce bubble trapping.

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