Emergence of collective modes, ecological collapse and directed percolation at the laminar-turbulence transition in pipe flow

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Deterministic classical mechanics of many particles in a box ➞ statistical mechanics
Deterministic classical mechanics of infinite number of particles in a box

= Navier-Stokes equations for a fluid

⇒ statistical mechanics
Deterministic classical mechanics of infinite number of particles in a box

\[ \nabla \cdot \mathbf{u} = 0 \]
\[ \rho \left( \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla p + \mu \nabla^2 \mathbf{u} \]

= Navier-Stokes equations for a fluid

\[ \rightarrow \] statistical mechanics
Transitional turbulence: puffs

- Reynolds’ original pipe turbulence (1883) reports on the transition
Precision measurement of turbulent transition

Q: will a puff survive to the end of the pipe?

Many repetitions $\Rightarrow$ survival probability $= P(Re, t)$

Pipe flow turbulence

Decaying single puff

laminar

metastable puffs

spatiotemporal intermittency

expanding slugs

Re

1775

2050

2500

Survival probability $P(\text{Re}, t) = e^{-\frac{t-t_0}{\tau(\text{Re})}}$

Pipe flow turbulence

Decaying single puff

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Spatiotemporal intermittency

expanding slugs

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Splitting probability

\[ 1 - P(Re, t) = e^{-\frac{t-t_0}{\tau(Re)}} \]

Pipe flow turbulence

Decaying single puff  →  laminar

metastable puffs  →  spatiotemporal intermittency

Splitting puffs  →  expanding slugs

Re  →  1775  →  2050  →  2500

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Pipe flow turbulence

Decaying single puff

Metastable puffs

Spatiotemporal intermittency

Expanding slugs

Re

Super-exponential scaling:

\[
\frac{\tau}{\tau_0} \sim \exp(\exp(Re))
\]

Puff lifetime

Mean time between split events

MODEL FOR METASTABLE TURBULENT PUFFS & SPATIOTEMPORAL INTERMITTENCY


Very complex behavior and we need to understand precisely what happens at the transition, and where the DP universality class comes from.
Logic of modeling phase transitions

Magnets

Electronic structure

Ising model

Landau theory

RG universality class (Ising universality class)
Logic of modeling phase transitions

Magnets

Electronic structure
→ Ising model
→ Landau theory
→ RG universality class (Ising universality class)

Turbulence

Kinetic theory
→ Navier-Stokes eqn
→ ?
Logic of modeling phase transitions

Magnets

Electronic structure
Ising model
Landau theory
RG universality class
(Ising universality class)

Turbulence

Kinetic theory
Navier-Stokes eqn

逻辑的建模相变理论
Identification of collective modes at the laminar-turbulent transition

To avoid technical approximations, we use DNS of Navier-Stokes
Predator-prey oscillations in pipe flow

Simulation based on the open source code by Ashley Willis: openpipeflow.org
What drives the zonal flow?

- Interaction in two fluid model
  - Turbulence, small-scale \((k>0)\)
  - Zonal flow, large-scale \((k=0,m=0)\): induced by turbulence and creates shear to suppress turbulence

1) Anisotropy of turbulence creates Reynolds stress which generates the mean velocity in azimuthal direction

\[
\partial_t \langle v_\theta \rangle = -\partial_r \langle (\tilde{v}_\theta \cdot \tilde{v}_r) \rangle - \mu \langle v_\theta \rangle
\]

2) Mean azimuthal velocity decreases the anisotropy of turbulence and thus suppress turbulence
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**Interaction in two fluid model**

- **Turbulence**, small-scale \((k>0)\)
- **Zonal flow**, large-scale \((k=0, m=0)\): induced by turbulence and creates shear to suppress turbulence
Population cycles in a predator-prey system

$\theta \approx \pi/2$ phase shift between prey and predator population

Persistent oscillations + Fluctuations

https://interstices.info/jcms/n_49876/des-especes-en-nombre
Derivation of predator-prey equations

Zonal flow -- Turbulence
Vacuum = Laminar flow

Zonal flow-turbulence

A = predator  B = prey
E = food/empty state

Predator-prey

\[ B + E \xrightarrow{b} B + B \]
\[ B + B \xrightarrow{c} B + E \]
\[ A + B \xrightarrow{p} A + A \]
\[ A + B \xrightarrow{p'} A + E \]
\[ B \xrightarrow{m} A \]
\[ A \xrightarrow{d_A} E \quad B \xrightarrow{d_B} E \]
Extinction/decay statistics for stochastic predator-prey systems
Pipe flow turbulence

Decaying single puff

laminar

metastable puffs

Re

1775

Splitting puffs

spatiotemporal intermittency

expanding slugs

2050

2500

Predator-prey model

nutrient only

metastable population

traveling fronts

expanding population

prey birth rate

0.02

0.05

0.08

Decaying population

Splitting populations
Pipe flow turbulence

Decaying single puff → laminar → metastable puffs

Splitting puffs → spatiotemporal intermittency → expanding slugs

Re
1775
2050
2500

Decaying population
Splitting populations

Linear stability of mean-field solutions

Nutrient only → metastable population → traveling fronts → expanding population

Prey birth rate
0.02
0.05
0.08
Puff splitting in predator-prey systems

Puff-splitting in predator-prey ecosystem in a pipe geometry

Puff-splitting in pipe turbulence
Avila et al., Science (2011)
Predator-prey vs. transitional turbulence

Prey lifetime

Mean time between population split events

T − ln (τ − τ₀/τ₀)

T − ln (τ − τ₀/τ₀) vs. Reynolds number

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Avila et al., Science 333, 192 (2011)

Predator-prey vs. transitional turbulence

Prey lifetime

Mean time between population split events

Turbulent puff lifetime

Mean time between puff split events

Extinction in Ecology = Death of Turbulence

Avila et al., Science 333, 192 (2011)
Roadmap: Universality class of laminar-turbulent transition

Universality class → (Classical) Turbulence → Two-fluid model → Predator-Prey

Direct Numerical Simulations of Navier-Stokes

(Boffetta and Ecke, 2012)

(Pearson Education, Inc., 2009)
Roadmap: Universality class of laminar-turbulent transition

Directed Percolation

Field Theory

Directed Percolation

Reggeon field theory
(Janssen, 1981)

Extinction transition
(Mobilia et al., 2007)

Predator-Prey

(Classical) Turbulence

Direct Numerical Simulations of Navier-Stokes

Two-fluid model

(Wikimedia Commons)

(Pearson Education, Inc., 2009)

(?)

(Wikimedia Commons)

(Boffetta and Ecke, 2012)
Directed percolation & the laminar-turbulent transition

- Turbulent regions can spontaneously relaminarize (go into an absorbing state).
- They can also contaminate their neighbourhood with turbulence. (Pomeau 1986)
Directed percolation transition

- A continuous phase transition occurs at $p_c$.

$$\rho \sim (p - p_c)^\beta$$
$$\xi_\perp \sim (p - p_c)^{-\nu_\perp}$$
$$\xi_\parallel \sim (p - p_c)^{-\nu_\parallel}$$
Directed percolation vs. transitional turbulence

Survival probability $P(Re, t) = e^{-\frac{t-t_0}{\tau(Re)}}$

Longest percolation path

Longest length of empty site

Turbulent puff lifetime

Decaying Turbulence:
- Injection ($L = 3300$)
- Hof et al. (2008)
- Kuik et al. (2010)
- Avila et al. (2010)

Shih and Goldenfeld (in preparation)
Directed percolation vs. transitional turbulence

Survival probability $P(\text{Re}, t) = e^{-\frac{t-t_0}{\tau(\text{Re})}}$

Directed percolation also has super-exponential lifetime!
Predator-prey & DP: connection?

• Near the laminar-turbulent transition, two important modes behave like predator-prey.

• Near the laminar-turbulent transition, lifetime statistics grow super-exponentially with Re, behaving like directed percolation.

• How can both descriptions be valid?
Universality class of predator-prey system near extinction

Basic individual processes in predator (A) and prey (B) system:

Death: \( B_i \xrightarrow{d_B} E_i \)  \( A_i \xrightarrow{d_A} E_i \)  \( B_i \xrightarrow{m} A_i \)

Birth: \( B_i + E_j \xrightarrow{b} \frac{b}{\langle ij \rangle} B_i + B_j \)  \( A_i + B_j \xrightarrow{p} \frac{p}{\langle ij \rangle} A_i + A_j \)

Diffusion: \( B_i + E_j \xrightarrow{D} \frac{D}{\langle ij \rangle} E_i + B_j \)  \( A_i + E_j \xrightarrow{D} \frac{D}{\langle ij \rangle} E_i + A_j \)

Carrying capacity: \( B_i + B_j \xrightarrow{c} \frac{c}{\langle ij \rangle} B_i + E_j \)
Universality class of predator-prey system near extinction

Near the transition to prey extinction, the prey (B) population is very small and no predator (A) can survive; A \sim 0.

\[ B_i \xrightarrow{d_B} E_i \]
\[ A_i \xrightarrow{d} E_i \]
\[ B_i \xrightarrow{m} A_i \]

\[ B_i + E_j \xrightarrow{b} \langle ij \rangle B_i + B_j \]
\[ A_i + B_j \xrightarrow{p} \langle ij \rangle A_i + A_j \]

\[ B_i + E_j \xrightarrow{D} \langle ij \rangle E_i + B_j \]
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**Birth**  \[ B_i + E_j \xrightarrow{b} \langle ij \rangle B_i + B_j \]

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Universality class of predator-prey system near extinction

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<table>
<thead>
<tr>
<th>Reaction Type</th>
<th>Equation</th>
<th>Diagram</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Death</td>
<td>$B_i \xrightarrow{d_B} E_i$</td>
<td>$t$</td>
<td>Annihilation</td>
</tr>
<tr>
<td>Birth</td>
<td>$B_i + E_j \xrightarrow{b_{ij}} B_i + B_j$</td>
<td>$t+1$</td>
<td>Decoagulation</td>
</tr>
<tr>
<td>Diffusion</td>
<td>$B_i + E_j \xrightarrow{D_{ij}} E_i + B_j$</td>
<td></td>
<td>Diffusion</td>
</tr>
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<td>Carrying capacity</td>
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<td></td>
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</table>
Universality class of predator-prey system near extinction

Near the transition to prey extinction, the prey (B) population is very small and no predator (A) can survive; $A \sim 0$.

Near the extinction transition, stochastic predator-prey dynamics reduces to directed percolation.
Summary: universality class of transitional turbulence

Directed Percolation

Reggeon field theory (Janssen, 1981)

Field Theory

Extinction transition (Mobilia et al., 2007)

Predator-Prey

(Classical) Turbulence

Direct Numerical Simulations of Navier-Stokes

Two-fluid model

(Wikimedia Commons)
Experimental evidence for directed percolation in transitional turbulence in different flow geometries
Turbulence and directed percolation

Fluid between concentric cylinders, outer one rotating

Turbulent patches

Position of turbulent patches changes in time

Lemoult et al., Nature Physics (2016)
Turbulence and directed percolation

Figure S1: a Schematic of experimental set up (not to scale) see text for details. b Image of turbulent spots and the conversion to a intensity time series (see text for details).

Lemoult et al., Nature Physics (2016)
Directed percolation in turbulence and ecology

Couette

Ecology
Experimental evidence for predator-prey dynamics in transitional turbulence
Universal predator-prey behavior in transitional turbulence experiments

- 2D magnetized electroconvection


- L-H mode transition in fusion plasmas in tokamak

  Estrada et al. *EPL* (2012)

http://alltheworldstokamaks.wordpress.com/gallery-of-external-views/kstar-completed/
Conclusion

• Transition to pipe turbulence is in the universality class of directed percolation, evidenced by:
  – Direct measurement of critical exponents and data collapse universal scaling functions in 1D Couette flow
  – DNS of stress-free Waleffe flow in 2D measures critical exponents and scaling functions

• How to derive universality class from hydrodynamics
  – Small-scale turbulence activates large-scale zonal flow which suppresses small-scale turbulence
  – Effective theory (“Landau theory”) is stochastic predator-prey ecosystem
  – Exact mapping: fluctuating predator-prey = Reggeon field theory = DP near extinction

• Super-exponential behavior of lifetime
  – Turbulence/DP/Predator-prey near extinction shows superexponential lifetime scaling for decay and splitting of puffs
Take-home message

- The Navier-Stokes equations quantitatively obey non-equilibrium statistical mechanics at the onset of turbulence
Turbulence is a life force. It is opportunity. Let's love turbulence and use it for change.

Lucky Numbers  34, 15, 28, 4, 19, 20
References

TRANSITIONAL TURBULENCE


• Hong-Yan Shih, Nigel Goldenfeld and collaborators. Statistical mechanics of puff-splitting in the transition to pipe turbulence. In preparation.

QUASI-CYCLES AND FLUCTUATION-INDUCED PREDATOR-PREY OSCILLATIONS

