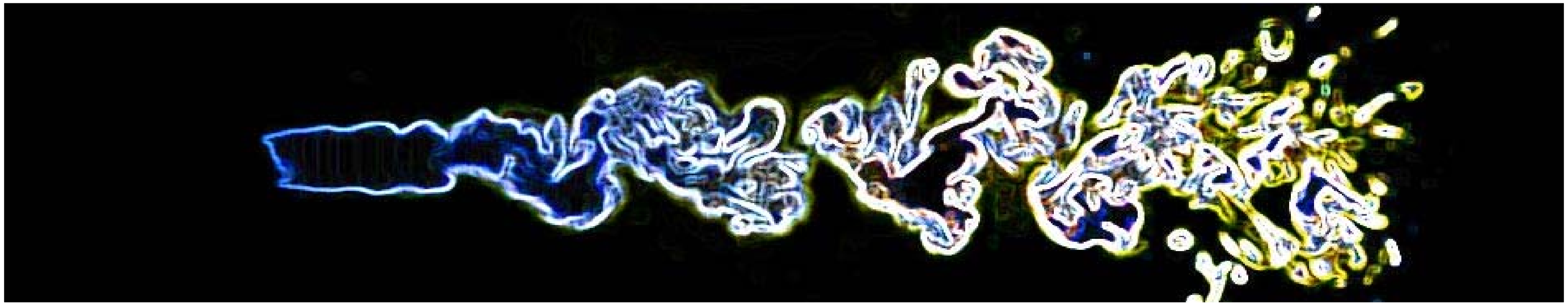
祝!!　Congraturations ！
日本流体力学会
五十周年記念
The 50th Anniversary
The Japan Society of Fluid Mechanics

50th Anniversary Symposium, Osaka University Hall, ***September 4th,*** **2018**



JSFM

Part I. Landscape of JSFM fifty years ago

Part II. New perspectives on mass conservation law and waves in fluid mechanics

by Tsutomu Kambe
Former Professor, University of Tokyo, Japan

Landscape
about 50 years ago

1962

Tokyo Metro. Gov. Area

2015

JR First Shinkansen

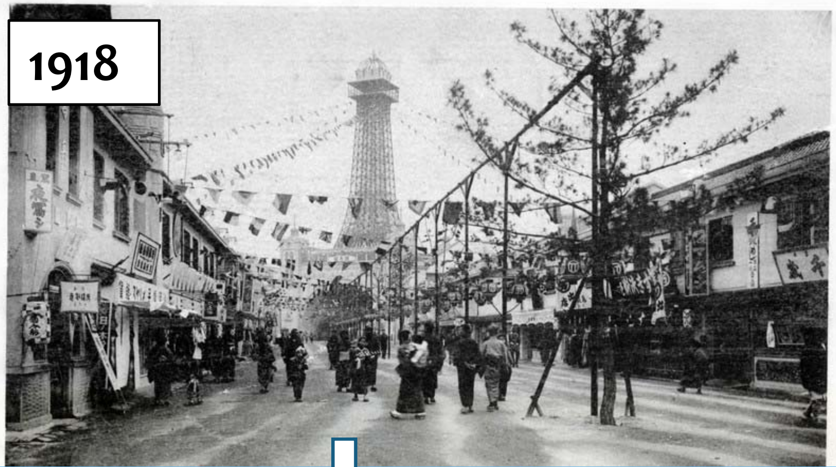
1964

2014

Landscapes

Osaka Shin-sekai: Tsuten-kaku Tower

1918



2017

Tokyo Shibuya

1952

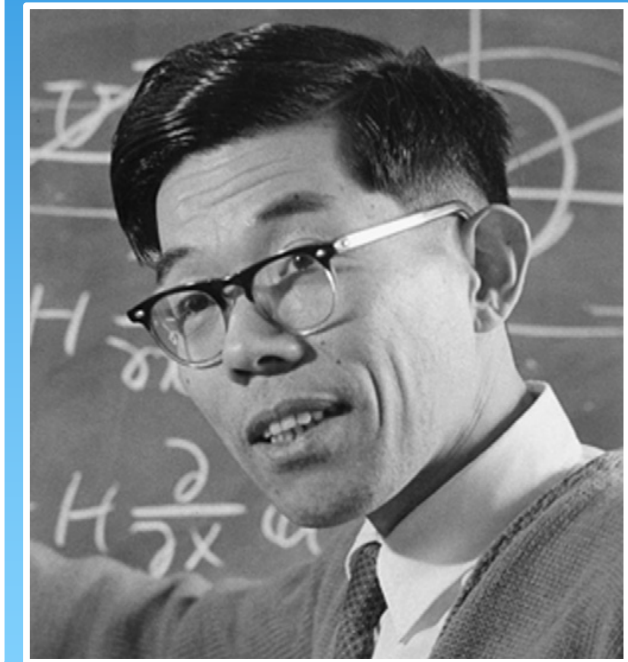
2017

pixta.jp – 8164989

Two top leaders in Japan around 1960

Prof. Itiro TANI
(about 53)

Prof. Isao IMAI
(about 45)

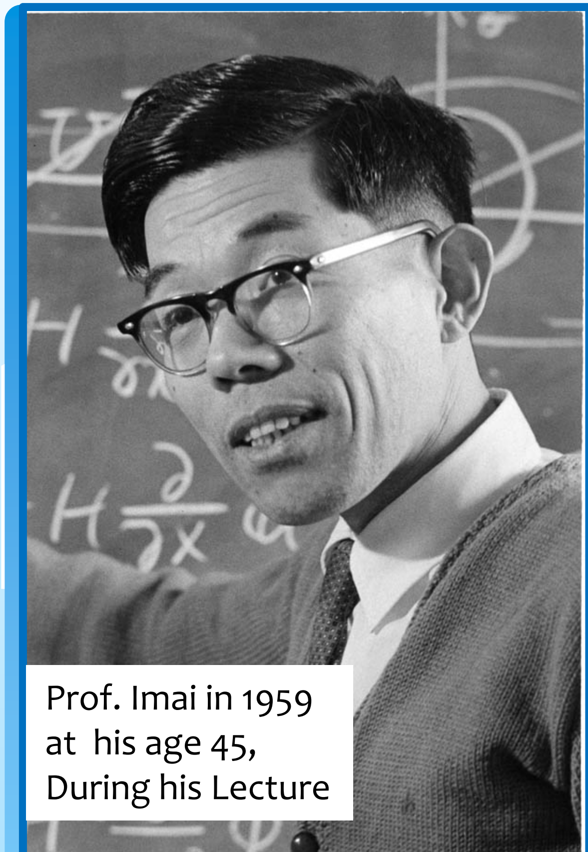


Part I. Pioneers of Fluid Mechanics in Japan at the start of JSFM

Let us see what they studied.

Pioneers at the start of JSFM (I)

(a) Laminar viscous flow around a circular cylinder



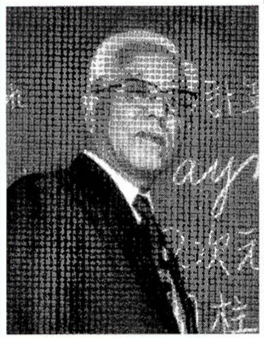
Prof. Imai in 1959
at his age 45,
During his Lecture

Imai's asymptotic expression of the stream function:

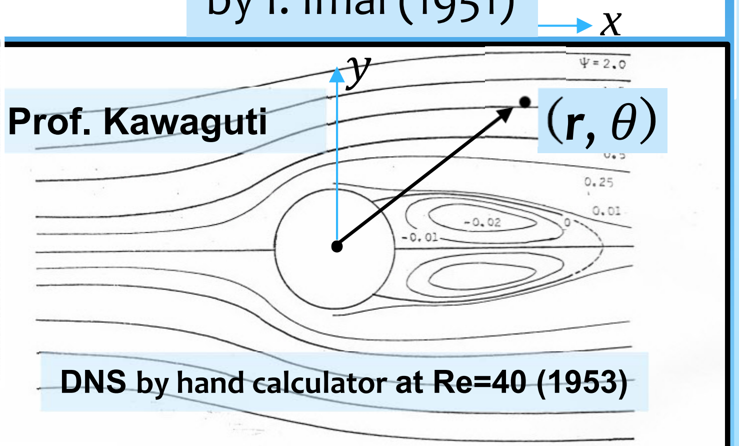
$$\frac{\Psi}{Ua} \rightarrow \quad y - \frac{C_D}{2}\left(1 - \frac{1}{\pi}\theta\right) + O\left(C_D\sqrt{R_e/r}\right)$$

as $r \rightarrow \infty$

by I. Imai (1951)



Prof. Kawaguti



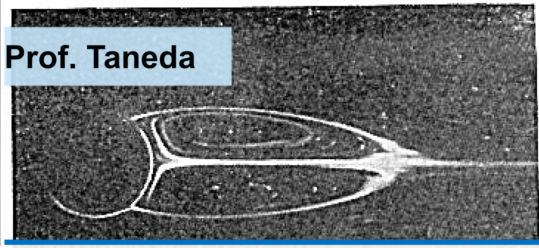
DNS by hand calculator at Re=40 (1953)

Fig. 4. Flow pattern (R=40).

The set of three works

- **provided a strong evidence that NS equation can describe steady laminar flows at moderate Reynolds numbers up to about 40,**
- **provided a stimulating hint for later development of *the method of Matched Asymptotic Expansions* by Proudman and Pearson (1957), Kaplun and Lagerstrom (1957).**

Prof. Taneda



Visualization in water channel with milk (1956)

Photo 2. (a) R=41.0.

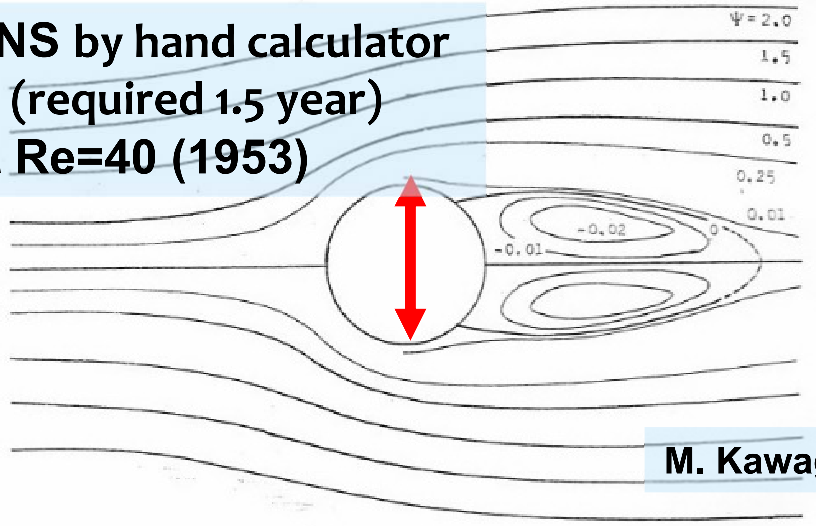
First successful collaborative works for laminar viscous flows around $Re \approx 40$

Asymptotic solution as $r \to \infty$

$$\Psi/(Ua) \to y - \frac{C_D}{2}\left(1 - \frac{1}{\pi}\theta\right) + \cdots$$

I. Imai (1951)

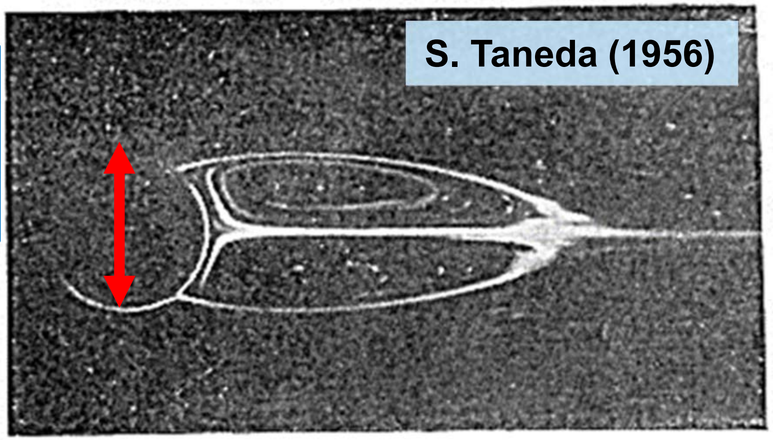
DNS by hand calculator (required 1.5 year) at Re=40 (1953)



M. Kawaguti (1953)

Fig. 4. Flow pattern (R=40).

Visualization in a water channel with milk (1956)



S. Taneda (1956)

Photo 2. (a) R=41.0.

Pioneers at the start of JSFM (II)

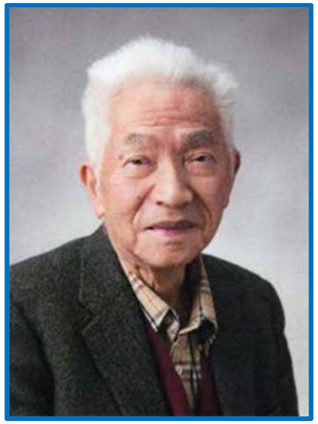
(b) Stability and turbulence

● 

Prof. T. Tatsumi

First study *of* turbulence in Japan with statistical theory

"The theory of decay process of incompressible isotropic turbulence" Proc. R. Soc. London A **239** (1957).

● 

Prof. H. Sato

Experimental study (first by using hot-wire): stability, transition and turbulence (Wind tunnel)

"The stability and transition of a two dimensional jet" *J. Fluid Mech.*, **7** (1960).

● **T. Tatsumi & T. Kakutani (1958):** Linear stability analysis of 2D Bickley jet

$R_c = 4.0, \qquad \alpha_c = 0.2$

● **T. Tatsumi & K. Gotoh (1960):** Linear stability of free shear layers

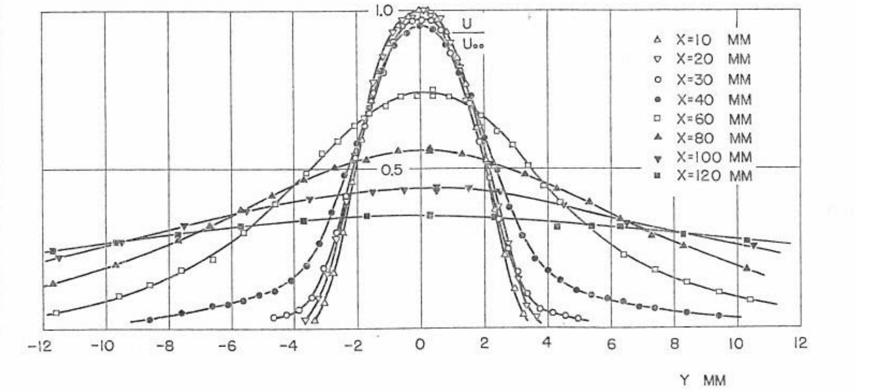


FIGURE 2. Mean-velocity distribution. SLIT 6 mm L, $U_{00} = 10{\cdot}0$ m/sec. X is measured from the slit.

Pioneers at the start of JSFM (III)

(c) *Streamwise vortices in boundary layer flows*

Late Prof. I. Tani

"Boundary-Layer Transition"

Annual Rev. Fluid Mech. vol.1 (1969)

- Tani, I. and Komoda, H.: Boundary-layer transition in the presence of streamwise vortices, J. Aerospace Sci., 29 (1962).
- Hino, M., Shikata, H. and Nakai, M.: Large eddies in stratified flows, Congr. Intern. Assoc. Hydraulic Res., XIIth (1967).

At the time of sixties, there was a gap between the observed phenomena of boundary layer transition to turbulence and the stability study of mainly linear analysis of 2D disturbances. Formation of 3D-disturbances is required for the flow transition to turbulence in the boundary layer.

Associated with the 3-dimensionality, there was an evidence of streamwise vortices in the boundary layers. This transition problem was reviewed by the *late* Professor Itiro Tani (1969), and studied by Tani & Komoda (1962), collaborating with the late Prof LSG Kovasznay staying in Tokyo. The vortices cause a redistribution of mean velocity field.

Later, the streak structure in boundary layer flows was interpreted by this mechanism.

Pioneers at the start of JSFM (IV)

(d) *Nonlinear waves*

Prof. H. Hasimoto

A soliton
on a vortex filament
J. Fluid Mech., 51 (1972)

Prof. A. Sakurai

On exact solution of the
blast wave problem,
J. Phys.Soc. Jpn. 10 (1955)

Fluid motion driven by locally concentrated vorticity can be described by *local-induction* law.
Hasimoto transformed **the law** into the *nonlinear Schrödinger equation*, and obtained a soliton solution of a deformed vortex filament.

A blast wave is usually generated as a shock caused by a powerful explosion such as asuper-nova or an atomic bomb.
Unlike the sound speed cs, the velocity U within the blast wave is not constant and always larger than the sound speed c_s.
Certain exact solutions of the blast wave problem were given by Sakurai for each of spherical, cylindrical and planar symmetry, citing G.I. Taylor:
Proc. R. Soc. London A ***201*** (1950).

1966: IUGG－IUTAM SYMPOSIUM ON BOUNDARY LAYERS AND TURBULENCE INCLUDING GEOPHYSICAL APPLICATION

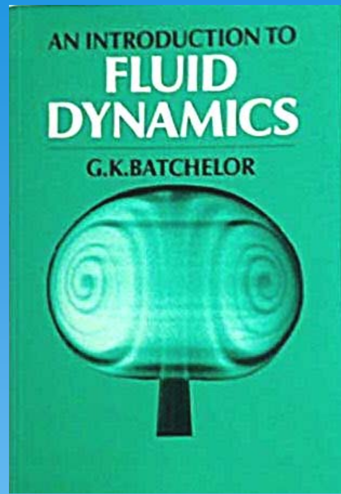
Symposium IUGG-IUTM, in 1966 (fifty-two years ago) at Kyoto

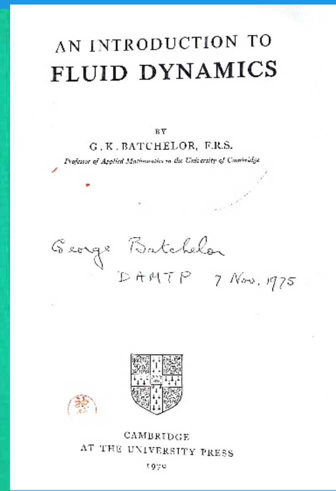
IUGG: International Union of Geodesy and Geophysics;
IUTAM: International Union of Theoretical and Applied Mechanics

In the photo, one can recognize (randomly):
H. Görtler, F.N. Frenkiel, I. Tani, A. Roshko, A.M. Yaglom, L.S.G. Kovasznay, J.O. Hinze, M.T. Landahl, S.I. Pai, P.S. Klebanoff, G.K. Batchelor, M.J. Lighthill, P.G. Saffman, L.G. Loitsianski, R. Betchov, D.J. ,Benney, J. Laufer, and many Japanese participants..

Batchelor and **Tanea**

- After the Kyoto conference, George Batchelor visited Taneda's laboratory at the *RIAM Institute, Kyushu Univ.*, and got interested in various visualization experiments carried out there by S. Taneda (1956), and also by Okabe & Inoue (1960, 61). He cited a number of photographs of their visualization in his textbook.
- Taneda was scouted by Prof. Hikoji Yamada to his laboratory in RIAM (*Research Institute for Applied Mechanics*).
- Batchelor Prize of IUTAM





Tani and **von Karman**

- In 1960, there was IUTA Symp. "MHD" at Williamsburg in USA, where there were several Japanese participants: **Tani, Imai, Tatsumi, Hasimoto** and others.

- There was Fluid Physics section at JPL of NASA administrated by Karman at Caltech. Besides its work in rocket propulsion, they received Japanese visitors: **Tatsumi, Sato, and Komoda.**

Part II. New perspectives on mass conservation law and waves in fluid mechanics

First of all:

We begin with the following recognition:

- **Conservation of energy** is related to **Time Translation Symmetry (Invariance).**
- Fundamental conservation equations of fluid mechanics are derived as **non-relativistic limit from the relativistic fluid mechanics.**
- From a single relativistic energy equation, we have two conservation equations in the non-relativistic limit $u^2/c^2 \rightarrow 0$:
  - Energy conservation equation of traditional form
  - Continuity equation
- A symmetry implies a conservation law (Noether, 1918).
- Then, we confront unusual situation.

What kind of **physical symmetry** implies the **Mass Conservation Law ?**

The **relativistic energy equation** can be written in the following way:

[Kambe (2017), citing Landau & Lifshitz (1987), Relativistic Fluid Dynamics (§ 133)]

$$[\,\partial_t\rho \;+ \mathrm{div}(\rho\boldsymbol{v})]\;\; c^2$$
$$+\Big[\,\partial_t\big(\rho(v^2/2+\epsilon)\big) + \mathrm{div}\big(\rho\boldsymbol{v}(v^2/2+h)\big)\Big]$$
$$+\;(\text{smaller order terms}) = 0$$

← Rest mass part of $O(c^2)$

← Flow energy part $O(u^2)$

We have

$$\partial_t\rho \;+\; \mathrm{div}(\rho\boldsymbol{v}) = 0,$$
$$\partial_t\big(\rho(v^2/2+\epsilon)\big) + \mathrm{div}\big(\rho\boldsymbol{v}(v^2/2+h)\big) = 0.$$

The textbook "Fluid Mechanics " of Landau & Lifshitz (1987) begins with the first section "**The equation of continuity**", deriving the equation,

$$\boldsymbol{\partial_t\rho \;+\; \mathrm{div}(\rho v) = 0},$$

mentioning just

one of the fundamental equations of fluid dynamics.

Symmetries imply conservation laws -- I

According to **Noether** (1918), published **100 years ago.**

Symmetry: ***Invariance property with respect to transformations.***

Lagrangian density: $\Lambda \equiv \Lambda\left(X^k, X_\mu^k\right) = \frac{1}{2} X_0^k X_0^k - \epsilon\left(X^k, X_l^k\right)$

Kinetic energy   Internal energy

(Ideal fluid)

$X^k = X^k(a^\mu)$: $a^0 = t \ (time)$; $a^k = X^k(t = 0)$ (**Lagrangian description**)

$\partial_\mu = \left(\partial_t, \partial_k\right)$, $k = 1,2,3$; $X_\mu^k = \partial_\mu X^k \equiv \partial X^k / \partial a^\mu$; $\mu = 0,1,2,3$; $k, l = 1,2,3$

Symmetry

Requiring ***invariance*** of Λ with respect to *local gauge transformation:* $X^k \to X^k + \delta X^k$

namely, $\delta\Lambda = 0$

$\Rightarrow$ Euler-Lagrange equation is derived: $[\mathcal{L}_{\text{eq}}] = 0$

Symmetries imply conservation laws -- I

According to **Noether** (1918), published **100 years ago.**

Lagrangian density: $\Lambda \equiv \Lambda(X^k, X^k_\mu) = \frac{1}{2} X^k_0 X^k_0 - \epsilon(X^k, X^k_l)$

(Ideal fluid)

(**Lagrangian description**)

$X^k_\mu = \partial_\mu X^k \equiv \partial X^k / \partial a^\mu ; \quad \mu = 0,1,2,3; \quad k, l = 1, 2, 3$

Incompressibility condition

$$\frac{\partial(X^k)}{\partial(a^l)} = \rho = 1$$

⇨ Euler-Lagrange equation

$$\boxed{\left[\mathcal{L}_{\text{eq}}\right]_k \equiv \partial_\mu \left(\frac{\partial \Lambda}{\partial X^k_\mu}\right) - \frac{\partial \Lambda}{\partial X^k} = 0}$$

Taking simple variation of Λ without vanishing boundary values

$$\boxed{\left[\partial_\mu \left(\frac{\partial \Lambda}{\partial X^k_\mu}\right) - \frac{\partial \Lambda}{\partial X^k}\right]} \delta X^k = [\partial_\nu T^\nu_\mu] \; \delta X^\mu$$

This is the Noether theorem.

Thus a symmetry $\left[\mathcal{L}_{\text{eq}}\right]_k = 0$ implies a conservation law: $\partial_\nu T^\nu_\mu = 0$

$\delta\Lambda = 0$ Is not assumed here, since $\delta\Lambda = \partial_\mu \Lambda \; \delta X^\mu$

where $$T^\nu_\mu = X^k_\mu \left(\frac{\partial \Lambda}{\partial X^k_\nu}\right) - \boxed{\Lambda \; \delta^\nu_\mu}$$: Energy-Momentum tensor

Fluid-Flow Energy –Momentum tensor

$$T_{\mathrm{f}}^{\alpha\beta} = \begin{pmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & \Pi^{11} & \Pi^{12} & \Pi^{13} \\ T^{20} & \Pi^{21} & \Pi^{22} & \Pi^{23} \\ T^{30} & \Pi^{31} & \Pi^{32} & \Pi^{33} \end{pmatrix}$$

$$T^{00} = \tfrac{1}{2}\rho v^2 + \rho\epsilon$$
$$T^{k0} = \rho\, v^k \left(\tfrac{v^2}{2} + h\right) = q_{\mathrm{f}}^k$$
$$T^{0k} = \rho\, v^k$$
$$\Pi^{ik} = \rho v^i v^k + p\,\delta^{ik}$$

Conservation of Energy and momentum:

$$\boldsymbol{\partial_\alpha T_{\mathrm{f}}^{\alpha\beta} = 0}$$

$$\partial_\alpha = (\partial_t, \partial_k), \quad \alpha = 0,1,2,3; \quad \partial_k = \partial/\partial x_k, \quad k = 1,2,3 \;.$$

$$\beta = 0: \quad \partial_t\left[\rho\left(\tfrac{1}{2}v^2 + \epsilon\right)\right] + \partial_k\left(q_{\mathrm{f}}^k\right) = 0.$$

Energy conservation

$$\beta = i = 1,2,3: \quad \partial_t\left[\rho\, v^i\right] + \partial_k \Pi^{ik} = 0.$$

Momentum conservation

Mass conservation law:

$$\partial_t \rho + \partial_k[\rho v^k] = 0.$$

This is valid, *a priori*.

Relativistic Energy–Momentum tensor for fluid-flow

$$\bar{T}^{\mu\nu}_{\mathrm{rel}} = \begin{pmatrix} \bar{T}^{00} & \bar{T}^{01} & \bar{T}^{02} & \bar{T}^{03} \\ \bar{T}^{10} & \Pi^{11} & \Pi^{12} & \Pi^{13} \\ \bar{T}^{20} & \Pi^{21} & \Pi^{22} & \Pi^{23} \\ \bar{T}^{30} & \Pi^{31} & \Pi^{32} & \Pi^{33} \end{pmatrix}$$

$$\begin{aligned} \bar{T}^{00} &= \tfrac{1}{2}\rho v^2 + \rho\epsilon + \boldsymbol{\rho\, c^2} \\ \bar{T}^{k0} &= \boldsymbol{c^{-1}} q_{\mathrm{f}}^k + \boldsymbol{c\rho v^k} \\ \bar{T}^{0k} &= \boldsymbol{c^{-1}} q_f^k + \boldsymbol{c\rho v^k} \\ \Pi^{ik} &= \rho v^i v^k + p\,\delta^{ik} \\ t &\Longrightarrow \tau = ct \end{aligned}$$

$$\boldsymbol{\partial_\mu \bar{T}^{\mu\nu}_{\mathrm{rel}} = 0}$$

$$\partial_\mu = \left(\tfrac{1}{c}\,\boldsymbol{\partial_{t,}}\, \partial_k\right), \qquad \partial_k = \partial/\partial x_k, \qquad k = 1,2,3$$

$\nu = 0$:

$$\Longrightarrow \quad \boldsymbol{c^2\left[\, \partial_t\rho + \partial_k\left(\rho v^k\right)\right]} + \left[\, \partial_t\{\rho\, (\tfrac{1}{2}v^2 + \epsilon)\} + \partial_k\{\rho\, v^k\, \left(\tfrac{v^2}{2} + h\right)\}\right] = 0$$

$O(c^2)$ $\qquad$ $O(u^2)$

This was neglected in Landau & Lifshitz (Fluid M.) because this is nothing but **the continuity equation.**

$$\Longrightarrow \begin{cases} \boxed{\boldsymbol{\partial_t\rho + \partial_k\left(\rho v^k\right) = 0}} \\ \boxed{\boldsymbol{\partial_t\{\rho\, (\tfrac{1}{2}v^2 + \epsilon)\} + \partial_k\left\{\rho\, v^k\, \left(\tfrac{v^2}{2} + h\right)\right\} = 0}} \end{cases}$$

Energy equation of **non-relativistic** fluid-flow

Relativistic momentumv equation

$$\nu \quad = 1,2,3 = i :$$

$$\frac{1}{c}\,\boldsymbol{\partial_t\left(c\rho v^k\right)} + \frac{1}{c}\,\boldsymbol{\partial_t}\left(\boldsymbol{c^{-1}}q_f^k\right) + \partial_k \Pi^{ik} = 0.$$

$$\Longrightarrow \qquad \partial_t\left[\,\rho\, v^i\right] + \partial_k \Pi^{ik} + \frac{1}{c^2}\,\boldsymbol{\partial_t\left(q_f^k\right)} = 0.$$

Mass conservation and Gauge symmetry I

- The mass conservation law is a law, **independent of coordinate frames.**

- To represent it, a ***frame-independent*** formulation using **differential forms** is most appropriate,

- and introduce **a new field** $\boldsymbol{a^{\beta}}$ of 4-vector potential

$$\boldsymbol{a_{\mu}(x^{\nu}) = (\phi_a, -a_1, -a_2, -a_3)} \qquad = g_{\mu\beta} a^{\beta},$$

$$g_{\alpha\beta} = \mathrm{diag}(+1, -1, -1, -1)$$

  in the 4-space-time $x^{\nu} = (t, x^1, x^2, x^3)$ of fluid flow.

- Let us define *one-form* $\mathbb{A}^{(1)}$ (a *gauge field*) by

$$\mathbb{A}^{(1)} \equiv a_{\mu}\, dx^{\mu} = \phi_a dt - a_1 dx^1 - a_2 dx^2 - a_3 dx^3$$

- This can lead to a ***gauge-invariant*** representation of governing equations.

Mass conservation law and gauge symmetry II

Taking external differential, two-form $\mathbb{F}^{(2)}$ is defined by

$$\mathbb{F}^{(2)} = d\mathbb{A}^{(1)} = \sum F_{\mu\nu}\, dx^{\mu} \wedge dx^{\nu}$$

$$F_{\mu\nu} = \begin{pmatrix} 0 & e_1 & e_2 & e_3 \\ -e_1 & 0 & -b_3 & b_2 \\ -e_2 & b_3 & 0 & -b_1 \\ -e_3 & -b_2 & b_1 & 0 \end{pmatrix}$$

where

$$\boldsymbol{e} = -\partial_t \boldsymbol{a} - \nabla\phi_a = (a_1. a_2, a_3)$$

$$\boldsymbol{b} = \nabla \times \boldsymbol{a} = (b_1. b_2, b_3)$$

Taking external differential again, First pair of Maxwell-type equations are given as **Identities derived from the differential form:**

$$\textbf{Eq. (I):} \quad d\mathbb{F}^{(2)} = d^2\mathbb{A}^{(1)} = 0 \quad \text{(Identity).}$$

$$\Longrightarrow \quad \boldsymbol{\nabla} \cdot \boldsymbol{b} = \boldsymbol{0}, \qquad \boldsymbol{\partial_t b} + \boldsymbol{\nabla} \times \boldsymbol{e} = \boldsymbol{0}$$

First pair of Maxwell-type equations.

Second pair of Maxwell-type equations

Second pair of Maxwell-type equations are derived from the Lagrangian by the variational principle, and represented as

Eq. (II): $$\partial_\mu G^{\mu\nu} = j^\nu, \qquad j^\nu = (\rho.\ j^1, j^2, j^3)$$

$$\Rightarrow \quad \boldsymbol{\nabla}\cdot\boldsymbol{d} = \rho, \qquad -\boldsymbol{\partial_t d} + \boldsymbol{\nabla}\times\boldsymbol{h} = \boldsymbol{j}.$$

where

$$G^{\mu\nu} = \begin{pmatrix} 0 & -d_1 & -d_2 & -d_3 \\ d_1 & 0 & -h_3 & h_2 \\ d_2 & h_3 & 0 & -h_1 \\ d_3 & -h_2 & h_1 & 0 \end{pmatrix}$$

$\boldsymbol{d} = \varepsilon\boldsymbol{e}, \qquad \boldsymbol{h} = \sigma^{-1}\boldsymbol{b}$

with $\boldsymbol{\varepsilon}$, $\boldsymbol{\sigma}$ parameters

Gauge transformation: $$\boldsymbol{a_\mu} \rightarrow \boldsymbol{a'_\mu} = \boldsymbol{a_\mu} - \boldsymbol{\partial_\mu \psi}$$

$$(G')^{\mu\nu} = \lambda\left[\partial_\mu a'_\nu - \partial_\nu a'_\mu\right] = \lambda\left[\partial_\mu(a_\nu - \cancel{\partial_\nu\psi}) - \partial_\nu(a_\mu - \cancel{\partial_\mu\psi})\right] = G^{\mu\nu}$$

Invariant !!

Lagrangian, Noether's theorem, Gauge invariance, and Mass conservation law

Lagrangian

$$\mathcal{L}_{\mathrm{w}} = \int d\mathcal{V}^4 \;\; \Lambda(a_\beta . \partial_\alpha a_\beta)$$

$$d\mathcal{V}^4 = dt \wedge dx^1 \wedge dx^2 \wedge dx^3$$

$$\Lambda = -\frac{1}{4} F_{\alpha\beta} G^{\alpha\beta} - j^\beta a_\beta$$

Variation: $\delta \mathcal{L}_{\mathrm{w}} = -\left[\partial_\nu \left(\frac{\partial \Lambda}{\partial(\partial_\nu a_\mu)}\right) - \frac{\partial \Lambda}{\partial a_\mu}\right] \delta a_\mu + \partial_\nu \left(\frac{\partial \Lambda}{\partial(\partial_\nu a_\mu} \delta a_\mu\right)$

Noether's theorem

$$\left[\partial_\nu \left(\frac{\partial \Lambda}{\partial(\partial_\nu a_\mu)}\right) - \frac{\partial \Lambda}{\partial a_\mu}\right] \delta a_\mu = [\partial_\nu T^\nu_\mu] \;\; \delta a_\mu$$

$$[\mathcal{L}_{\mathrm{eq}}] = \left[\partial_\nu \left(\frac{\partial \Lambda}{\partial(\partial_\nu a_\mu)}\right) - \frac{\partial \Lambda}{\partial a_\mu}\right]$$

$$T^\nu_\mu = \partial_\mu a_\lambda \left(\frac{\partial \Lambda}{\partial(\partial_\nu a_\lambda)}\right) - \Lambda \;\delta^\nu_\mu$$

$$[\mathcal{L}_{\mathrm{eq}}] = 0 \quad \Longrightarrow \quad \partial_\nu T^\nu_\mu = 0.$$

We have the Noether's theorem for this Mawell-type system too.

The theorem does not necessarily apply to systems that cannot be modeled with a Lagrangian alone.

Gauge invariance implies the mass conservation equation

Lagrangian

$$\mathcal{L}_{\mathrm{w}} = \int d\mathcal{V}^4 \;\; \Lambda(a_\beta . \partial_\alpha a_\beta)$$

$$\Lambda = -\frac{1}{4} F_{\alpha\beta} G^{\alpha\beta} - j^\beta a_\beta$$
$$= \tfrac{1}{2}\varepsilon\,(\boldsymbol{e},\boldsymbol{e}) - \tfrac{1}{2}\mu^{-1}\,(\boldsymbol{b},\boldsymbol{b}) - \rho\phi_a + j^k a_k$$

Metric tensor:

$$g_{\mu\nu} = \mathrm{diag}(+1,-1,-1,-1)$$
$$a_\beta = g_{\beta\alpha}\, a^\alpha$$

Another form of **equivalent** variational formulation:

$$\delta\Lambda\, d\mathcal{V}^4 = (M_{II}{}^{\mu}\, g_{\mu\nu}\, \delta a^\nu\,)\, d\mathcal{V}^4$$

$$d\mathcal{V}^4 = dt \wedge dx^1 \wedge dx^2 \wedge dx^3$$

$$\delta a^\nu = (\quad \delta\phi_a, \qquad \delta\boldsymbol{a})$$

$$M_{II}{}^{\mu} = (\,\nabla\cdot\boldsymbol{d} - \rho, \qquad -\partial_t\boldsymbol{d} + \nabla\times\boldsymbol{h} - \boldsymbol{j}\,)$$

$M_{II}{}^{\mu} = 0$ for arbitrary variation δa_μ $\Longrightarrow$ $M_{II}{}^{\mu} = 0.$ (Maxwell's second pair)

$$\nabla\cdot\boldsymbol{d} = \rho, \qquad -\partial_t\boldsymbol{d} + \nabla\times\boldsymbol{h} = \boldsymbol{j}$$

Next, we consider its Gauge Invariance.

Gauge invariance implies the mass conservation equation

Lagrangian

$$\mathcal{L}_{\mathrm{w}} = \int d\mathcal{V}^4 \;\; \Lambda(a_\beta . \partial_\alpha a_\beta)$$

$$\Lambda = \tfrac{1}{2}\varepsilon\,(\boldsymbol{e},\boldsymbol{e}) - \tfrac{1}{2}\mu^{-1}\,(\boldsymbol{b},\boldsymbol{b}) - \rho\phi_a + j^k a_k$$

Metric tensor:

$$g_{\mu\nu} = \mathrm{diag}(+1,-1,-1,-1)$$

$$a_\beta = g_{\beta\alpha}\, a^\alpha$$

$$d\mathcal{V}^4 = dt \wedge dx^1 \wedge dx^2 \wedge dx^3$$

Variation:

$$\delta\Lambda\, d\mathcal{V}^4 = (M_{II}^{\ \mu}\, g_{\mu\nu}\, \boldsymbol{\delta a^\nu}\,)\, d\mathcal{V}^4$$

$$M_{II}^{\ \mu} = (\,\nabla\cdot\boldsymbol{d} - \rho, \qquad -\partial_t\boldsymbol{d} + \nabla\times\boldsymbol{h} - \boldsymbol{j}\,)$$

Variation of the gauge field a_μ:

$$\boldsymbol{\delta_{\mathrm{G}} a_\mu = -\partial_\mu \psi}$$

for arbitrary scalar field ψ

$$\boldsymbol{\delta_G \Lambda}\, d\mathcal{V}^4 = M_{\mathrm{II}}^{\mu}\;\; \boldsymbol{\delta_G a_\mu}\;\; d\mathcal{V}^4 = \left(\boldsymbol{\partial_\mu M_{II}^{\mu}}\right)\psi\, d\mathcal{V}^4 - \text{(vanishing boundary values)}$$

Gauge Invariance

$\boldsymbol{\delta_G \Lambda}$ **=0** $\Rightarrow$

Mass conservation law

$$\boldsymbol{\partial_\mu M_{II}^{\mu} = -(\partial_t \rho + \nabla\cdot j) = 0}$$

Electromagnetism from Clasical Theory of Fields: Landau and Lifshitz (1975), at a footnote in § 18 (Gauge invariance)

The *gauge invariance* is related to the assumed constancy of the electric charge *e*. Thus, *the **gauge invariance of the equations of electrodynamics*** and ***the conservation of charge*** are closely related to one another.
Commented already 40 years ago !!

A transformation which alters non-observable properties of fields (e.g. potentials) without changing the physically-meaningful measurable magnitudes (e.g. intensities).

Our system of Fluid Flows:

We may rephrase this to our system of Fluid Flows as follows.

The **new fields** $(\boldsymbol{e}, \boldsymbol{b})$ are derived from the gauge potential $a_\mu(x^\nu)$.

The ***gauge-invariance* of $(\boldsymbol{e}, \boldsymbol{b})$ field implies the mass conservation law.**

This is another example of Noether's theorem.

Conversely, the Mass Conservation law **implies**

Existence of new gauge-invariant fields $(\boldsymbol{e}, \boldsymbol{b})$.

This is one of the propositions of the present work.

Total Field = (Fluid-flow field F) + (Wavy Field W)

According to a general principle of theoretical physics, the combined field is defined by linear combination of Lagrangians describing each constituent field.

T. Kambe (2017): ***New scenario of turbulence theory and wall-bounded turbulence: theoretical significance"*** ***Geophys. Astrophys. Fluid Dyn.*** **Vol.111, 448-507.**

Defining the Energy-Momentum tensor of the total system with $T^{\alpha\beta}_{\mathrm{fw}} = T^{\alpha\beta}_{\mathrm{f}} + T^{\alpha\beta}_{\mathrm{w}}$ the system is governed by

$$\partial_\alpha T^{\alpha\beta}_{\mathrm{fw}} = \partial_\alpha T^{\alpha\beta}_{\mathrm{f}} + \partial_\alpha T^{\alpha\beta}_{\mathrm{w}} = 0$$

$\beta = 0$

$$\partial_t\left[\rho\left(\tfrac{1}{2}v^2 + \epsilon\right) + \tilde{e}_w\right] + \nabla\cdot(\boldsymbol{q}_{\mathrm{f}} + \boldsymbol{q}_{\mathrm{w}}) = 0$$

Total energy densities

Energy fluxes

$\beta = 1,2,3$

$$\partial_t[\rho\boldsymbol{v} + \boldsymbol{g}] + \nabla\cdot\left(\bar{\bar{\Pi}} + \bar{\bar{M}}\right) = 0$$

Total momentum densities

Stress tensors

Momentum equations for each component

F - Field: $\partial_t(\rho \boldsymbol{v}) + \nabla \cdot \Pi = \boxed{\boldsymbol{F}_{\mathrm{L}}[\boldsymbol{a}]}$

W - Field: $\partial_t \boldsymbol{g} + \nabla \cdot M = \boxed{-\boldsymbol{F}_{\mathrm{L}}[\boldsymbol{a}]}$

$$\boxed{\boldsymbol{F}_{\mathrm{L}}[\boldsymbol{a}]} = \rho \boldsymbol{e} + \boldsymbol{j} \times \boldsymbol{b} = \rho \boldsymbol{f}_{\mathrm{L}}$$

$$\Pi_{ij} = \rho v_i v_j + p\, \delta_{ij}$$

$$M_{ij} = -\left(e_i d_j + h_i b_j\right) + \tfrac{1}{2}(\boldsymbol{e} \cdot \boldsymbol{d} + \boldsymbol{h} \cdot \boldsymbol{b})\delta_{ij}$$

$$\boldsymbol{f}_{\mathrm{L}} = \boldsymbol{e} + \rho^{-1} \boldsymbol{j} \times \boldsymbol{b}$$

W-field acts on F-field with a force $\boldsymbol{F}_{\mathrm{L}}[\boldsymbol{a}]$, While the F-field reacts backto the W-field with a reaction force $-\boldsymbol{F}_{\mathrm{L}}[\boldsymbol{a}]$.

Energy equations for each component

F - Field: $\partial_t\left[\rho\left(\frac{1}{2}v^2+\epsilon\right)\right] + \nabla\cdot\boldsymbol{q}_{\mathrm{f}} = \boldsymbol{j}\cdot\boldsymbol{e}$

W - Field: $\partial_t\tilde{e}_w + \nabla\cdot q_w = \boxed{-\boldsymbol{j}\cdot\boldsymbol{e}}$

When the W-field loses energy $\boldsymbol{j}\cdot\boldsymbol{e}$ $(>\mathbf{0})$, then the F-field gains the same amount of energy. If the lost energy was dissipative, the heat energy should be absorbed as internal energy、resulting in *Entropy* increase.

$\boldsymbol{q}_{\mathrm{f}} = \rho v\left(\frac{1}{2}v^2 + h\right)$

$\boldsymbol{q}_{\mathrm{w}} = \boldsymbol{e}\times\boldsymbol{h}$

$\tilde{e}_{\mathrm{w}} = \frac{1}{2}(\boldsymbol{e}\cdot\boldsymbol{d} + \boldsymbol{h}\cdot\boldsymbol{b})$

$\boldsymbol{g} = \boldsymbol{d}\times\boldsymbol{b}$

Current flux and assumed dissipation

Energy flux was a linear combination of $\boldsymbol{q}_{\mathrm{f}}$ and $\boldsymbol{q}_{\mathrm{w}}$.
Likewise, current flux is represented as $\boldsymbol{j} = \boldsymbol{j}_{\mathrm{f}} + \boldsymbol{j}_{\mathrm{w}}$.

$$\boldsymbol{f}_{\mathrm{L}} = \boldsymbol{e} + \rho^{-1}\boldsymbol{j} \times \boldsymbol{b}$$

$$\boldsymbol{j}_{\mathrm{f}} = \rho \boldsymbol{v} \qquad \boldsymbol{j}_{w} = \sigma \boldsymbol{f}_{\mathrm{L}} \approx \boldsymbol{j}_{\boldsymbol{D}} = \sigma \boldsymbol{e} \quad (\text{assumed})$$

Rate of dissipation due to W-field: WD-effect

$$Q_w = j_w \cdot e \quad \Rightarrow \quad Q_{\mathrm{wD}} = |j_d|^2/\sigma > 0$$

Entropy $\boldsymbol{s}$ increases by the heat released:

$$\rho T \frac{Ds}{Dt} = Q_{\mathrm{vis}} + Q_{\mathrm{wD}}$$

Viscous dissipation:

$$Q_{\mathrm{vis}} = \rho \, \frac{\nu_m}{2} \left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i} \right)^2 \sim \rho \nu_m \left(\frac{u}{d} \right)^2$$

for $\frac{\partial v_i}{\partial x_i} = 0$

Dissipation due to WD-effect

$$Q_{\mathrm{wD}} = j_w \cdot e \sim \rho \, \nu_D \left(\frac{u_D}{d_w} \right)^2$$

$\nu_m \sim c_s l_m$: molecular viscosity

$\nu_D \sim c_t d_w$ like an eddy viscosity

Conceptual diagram, showing

streaky structure of wall-bounded turbulence is a dissipative structure

Main basic wall flow $\boldsymbol{v}$

Kambe (2017)

wave equation

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\boldsymbol{e} = \mu\,\partial_t(\rho\boldsymbol{v}) + \mu\,\sigma\,\partial_t\boldsymbol{e}$$

energy equation

$$\frac{\partial}{\partial t}w \;+\; \mathrm{div}\,(\boldsymbol{e}\times\boldsymbol{h}) \;=\; -\,\boldsymbol{e}\cdot\boldsymbol{j}_c - \,\boldsymbol{e}\cdot\boldsymbol{j}_d$$

Energy

Unstable Streaky structure

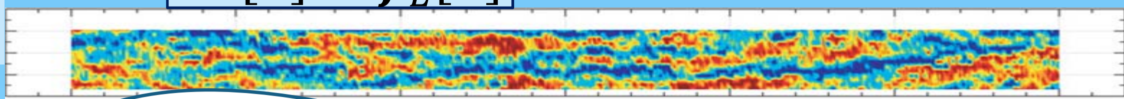
Energy

Wavy field $\boldsymbol{a}$

Wavy disturbance

$$\mathrm{NS}[\boldsymbol{v}] = \boldsymbol{f}_L[\boldsymbol{a}]$$

Fluid-Joule dissipation



Dissipative structure

heat

There is energy flux through the structure from main flow to heat, whereas the structure is maintained.

Color-picture inset: Thanks to **Monty, Stewart, Williams, and Chong**: *"Large-scale features in turbulent pipe and channel flow"*, JFM. **589 (2010),** 147-156.

Comparison of WD-effect and viscosity
Rate of Dissipation

$$Q_{\mathrm{wD}} = j_w \cdot e \sim \frac{1}{\sigma}|j_D|^2 = \frac{1}{\sigma}\rho^2 {u_D}^2 \sim \rho\,\boxed{(c_t d_w)}\left(\frac{u_D}{d_w}\right)^2$$

$\nu_D \sim c_t d_w$

$\boldsymbol{j_w} \approx \boldsymbol{j_D} = \rho u_D \sim \sigma e\,,\quad e \sim a_D/\tau_D,$

$\sigma \sim \rho\tau_D \sim \rho\, d_w/c_t$

$\rho\, u_D \sim \sigma\, e \sim \sigma\, a_D/\tau_D \sim \sigma\, u_D/\tau_D$

$d_w \sim c_t\tau_D$

c_t : Wave speed (transversal)

d_w : Wave's damping distance

τ_D : Wave's damping time

Eddy viscosity of WD-effect

$$\nu_D \approx c_t d_w \sim \left(50\ \frac{cm}{sec}\right) \times (60\ cm) \sim 10^3\ cgs$$

Estimated from the experiment by ***Kim & Adrian*****, Phys. Fluids, 11 (1999) 417, for pipe turbulence at** $\boldsymbol{Re} \approx \mathbf{10^5}$.

Molecular viscosity

$$\nu_m \approx c_s l_m$$

(sound speed)×(mean free path of Molecules)

$$\sim \left(3\times 10^4\ \frac{cm}{sec}\right) \times (7\times 10^{-6} cm) \sim 10^{-1}\ cgs$$

New Field

- **Mass Conservation Law** $(\partial_t \boldsymbol{\rho} + \boldsymbol{\nabla} \cdot \boldsymbol{j} = 0)$ is represented by new ***Gauge-invariant Fields*** $(\boldsymbol{e}, \boldsymbol{b})$ as

$$\rho = \nabla \cdot (\epsilon \boldsymbol{e}), \qquad \boldsymbol{j} = -\partial_t (\epsilon \boldsymbol{e}) + \nabla \times (\mu^{-1} \boldsymbol{b}).$$

- The Noether's thorem implies the mass Conservation law.
- Conversely, the mass conservation law implies

Existence of new gauge-invariant fields $(\boldsymbol{e}, \boldsymbol{b})$.

- The fluid-flow field is acted on by the gauge field with a Lorentz-like force:

$$\boldsymbol{F}_{\mathrm{L}}[\,\boldsymbol{a}\,] = \rho \boldsymbol{e} + \boldsymbol{j} \times \boldsymbol{b}, \qquad \boldsymbol{e} = -\boldsymbol{\nabla} \phi_a - \partial_t \boldsymbol{a}$$

Gravitational force is implied by the expression $\nabla^2 \phi_a = \mathcal{C}\, \rho$, with $\mathcal{C}$ a constant. and represented by $\boldsymbol{F}_{\mathrm{L}}[\,\boldsymbol{a}\,] = -\rho \boldsymbol{\nabla} \phi_a + \ldots$.

- The energy equation can be generalized to include a dissipation effect by a dissipative mass-current, $\boldsymbol{j_D} = \sigma \boldsymbol{e}$, which enables much higher rate of dissipation than the viscous effect.

Concluding Remarks

In Japan, more than fifty years ago, the study field of Fluid Mechanics did not have a fixed position in the academic community.

- In Physics community, it was regarded as Classical and Applied Mathematics (sometimes as macroscopic, or phenomenology), while in the Engineering community, regarded as being too theoretical, or too mathematical.
- This situation was one of the motivations to establish our society JSFM.
- However now, the present speaker believe personally that the Fluid Mechanics should not be called as *classical*, but it is **one of the simplest models of field theory** of physical systems, because Fluid Mechanics can be described by Lagrangian functionals which consistent with the gauge theory of theoretical physics.
- Fluid Mechanics is not only based on the field theory of Physics on the fundamental level, but the fields covered by Fluid Mechanics are diverse: Geo-spheres, Cosmic-space, engineering technologies, bio-spheres, nano-sphere. In particular, atmosphere, ocean, climate, and many others.
- New age gives us new challenging problems.

Thank you very much for your kind attention !!

Typhoon No.21
September 1st , 2018

Finish

September 4^{th}, 2018