# FLOW PATTERNS PAST TWO NEARBY CIRCULAR CYLINDERS 

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#### Abstract

In this investigation，flow patterns past two identical nearby circular cylinders at $R e=100$ are numerically studied as a basic model for laminar wake interaction．An immersed boundary method is employed for effective treatment of the cylinders on a Cartesian grid system．We consider all possible arrangements of the two circular cylinders in terms of the distance between them and the inclination angle of the line connecting their centers with respect to the main flow direction．It is found that eight patterns of distinct flow characteristics are identified by vorticity contours and streamlines．Collecting all the results obtained，we propose the flow－pattern diagram（＂map＂）for the two cylinders to provide an overall picture on the wake interaction．The perfect geometrical symmetry implied in the flow configuration allows one to use this diagram to distinguish flow pattern past two identical circular cylinders arbitrarily positioned in physical space with respect to the main flow direction．


## 1．Introduction

Cross－flow past a group of cylinders is often found in practical engineering applications．Flow characteristics past each cylinder are affected by its neighbors via wake interaction，resulting in alteration of the overall flow pattern．Therefore，flow pattern past multiple cylinders heavily depends on their relative positions with respect to the main－flow direction．Consequently，vortex－shedding frequency of the individual cylinder is accordingly determined，and serves as an important factor in generation of flow noise［1］．Being motivated by this，many researchers have been involved in studying wake interaction between two circular cylinders of equal diameter immersed in a cross freestream as a basic wake－interaction model［2，3，4，5］．In spite of the numerous studies carried out so far，the case where the two cylinders are placed in a staggered position has been rarely studied，especially in the laminar flow regime．In this investigation，flow patterns past two nearby circular cylinders of equal diameter immersed in the cross－flow at $R e=100$ ，based on the freestream velocity $(U)$ and the cylinder diameter $(D)$ ，were numerically studied as a basic model for laminar wake interaction．An immersed boundary method［6］was employed for effective treatment of the cylinders on a Cartesian grid system．We consider all possible arrangements of the two circular cylinders in terms of the distance between them and the inclination angle of the line connecting their centers with respect to the main flow direction．

## 2．Formulation and Numerical Methodology

The current investigation requires a parametric study where numerous numerical simulations must be performed with various values of the streamwise center distance $(L)$ and the vertical center distance $(T)$ ．See Fig． 1（a）．The governing equations for two－dimensional incompressible flow， modified for the immersed boundary method［6］，are as follows；

$$
\begin{align*}
& \frac{\partial u_{j}}{\partial x_{j}}-q=0 \quad j=1,2  \tag{1}\\
& \frac{\partial u_{i}}{\partial t}+\frac{\partial u_{i} \partial u_{j}}{\partial x_{j}}=-\frac{\partial p}{\partial x_{i}}+\frac{1}{\operatorname{Re}} \frac{\partial^{2} u_{i}}{\partial x_{j} \partial x_{j}}+f_{i} \tag{2}
\end{align*}
$$

where $u_{i}$（or $u, v$ ），$p, q$ and $f_{i}$ represent velocity component in $x_{i}$（or $x, y$ ） direction，pressure，mass source／sink，and momentum forcing， respectively．All the physical variables except $p$ are nondimensionalized by $U$ and $D$ ；pressure is nondimensionalized by far－field pressure（ $p_{\infty}$ ）and the dynamic pressure．The governing equations were discretized using a finite－volume method in a nonuniform staggered Cartesian grid system． Spatial discretization is second－order accurate．A hybrid scheme is used
for time advancement；nonlinear terms are explicitly advanced by a third－order Runge－Kutta scheme，and the other terms are implicitly advanced by the Crank－Nicolson method．A fractional step method［7］ was employed to decouple the continuity and momentum equations．The Poisson equation resulted from the second stage of the fractional step method was solved by a multigrid method．For detailed description of the numerical method used in the current investigation，see Yang and Ferziger［8］．


Figure 1．Physical configuration：（a）staggered position of two circular cylinders，（b）locations of the center of the surrounding circular cylinder，indicated by dots．

The main cylinder（MC）is fixed at the origin of the coordinate system， and the downstream cylinder（hereafter，called＂surrounding cylinder＂， SC ）is placed on various locations relative to MC，which are represented by dots in Fig．1（b）．The total number of cases computed is 208．The entire computational domain was defined as $-35 D \leq x \leq 35 D$ ，and $-50 D \leq y \leq 50 D$ ．For each cylinder， $32 \times 32$ uniform grid cells in $x$ and $y$ directions，respectively，were allocated，and uniform grid cells of the same cell size as in the cylinder region were employed between the cylinders．In the other region of the domain，nonuniform grid cells were used．The numerical resolution was determined by a grid－refinement study to ensure grid－independency．The maximum number of grid cells allocated were $480 \times 352$ ．No－slip condition was imposed on the cylinder surfaces；a Dirichlet boundary condition $(u=U, v=0)$ was used on the inlet boundary of the computational domain，while a convective boundary condition was employed at the outlet．A slip boundary condition $(\partial u / \partial y=0$ ， $\nu=0$ ）was imposed on the other boundaries．

## 3．Results and Discussion

3． 1 Classification of Flow Pattern
In the present investigation，we classified types of flow pattern based on streamlines and contours of spanwise vorticity $\left(\omega_{z}\right)$ ．Spanwise vorticity Copyright © 2007 by JSFM
is defined as $0.5(\partial v / \partial x-\partial u / \partial y)$ ．
Base Bleed pattern is noticed when $(L / D, T / D)=(0.0,1.25),(0.25,1.0)$ ， $(0.25,1.25),(0.5,1.0),(0.75,0.75)$ ．Even though the distance between the two cylinders is quite short，the flow through the＂gap＂is significant owing to the large $\theta$ ．Low pressure region alternates up and down behind MC；the gap flow also alternates accordingly．The short distance between the cylinders prevents＂individual＂shedding；the vortices are shed in a fashion similar to that behind a single object．

Biased Base Bleed pattern is found when $(L / D, T / D)=(1.0,0.25),(1.0$ ， $0.5)$ ．The gap between the cylinders is very narrow like the BB pattern， but $\theta$ is small unlike the BB pattern．The flow through the gap is always heading downwards due to the small $\theta$ ，while in the BB pattern，the direction of the gap flow is alternating up and down．Like in the BB pattern，vortices are shed as if the two cylinders were just one object．
Shear Layer Reattachment pattern is observed approximately when $1.25 \leq L / D \leq 4.0, T / D \leq 0.75$ ；in this range，$\theta$ is less than $20^{\circ}$ ．The shear layer formed above MC reattaches on SC ．A lower portion of the shear layer is deflected by SC，and together with the shear layer formed below MC ，surrounds the recirculation region behind MC．It is also noticed that vortex shedding from MC has been completely suppressed by SC，and vortex shedding from SC is somewhat delayed．This is one of the important features of SLR pattern．Sharman et al．［3］reported that in tandem arrangements of the two cylinders，vortex shedding from MC resumes at a certain＂critical＂distance between them which is in the range of $3.75 \leq L / D \leq 4.0$ depending on Re．

Induced Separation pattern is noticed approximately when $1.0 \leq L / D \leq 3.5,0.75 \leq T / D \leq 1.75$ ．In this region，$\theta$ is relatively large compared to that of SLR pattern．The shear layer formed above MC does not reattach on SC，but is deflected by SC．Unlike SLR pattern，vortices are shed from MC in IS pattern．

Vortex Impingement pattern appears in the range of approximately $L / D \geq 4.0, T / D \leq 1.75$ ，where the two cylinders are far apart（ $R / D \geq 4.0$ ） and $\theta$ is less than $23^{\circ}$ ．When the two cylinders are placed in tandem with a short distance between them（ $L / D \leq 3.75$ for $R e=100$ ），SC suppresses the vortex shedding of MC［3］．With a larger distance，however，SC fails to suppress the vortex shedding of MC，and vortices are shed from both cylinders．Vortices of positive and negative signs，shed from MC in an alternating manner，impinge upon SC ．However，when the two cylinders are slightly＂off－tandem＂（say，$T / D \geq 0.25$ ），only the vortices which developed in the upper shear layer of MC impinge upon SC while the vortices in the opposite shear layer simply pass SC without impingement．
Flip Flopping pattern is observed in the range of approximately $L / D \leq 2.5,1.5 \leq T / D \leq 2.0$ where the distance between the two cylinders $(1.34 \leq R / D \leq 3.20)$ is somewhat larger than those in BB and BBB patterns，and $\theta \geq 45^{\circ}$ ．In this pattern，both cylinders shed vortices， yielding two pairs of counter－rotating vortices．Unlike BB or BBB pattern， however，FF pattern is irregular in time，being consistent with Kang＇s finding［2］．He reported that in side－by－side arrangements of two cylinders， flow pattern is irregular when $1.4 \leq T / D \leq 2.2$ ．

Modulated Periodic pattern occurs in the range of approximately $0.5 \leq L / D \leq 3.0, T / D \geq 2.5$ where the two cylinders are far apart （ $2.69 \leq R / D \leq 5.59$ ），and approximately $\theta \geq 45^{\circ}$ ．Combination of the large distance and the relatively high inclination angle yields weak interaction between the two wakes，resulting in slight modulation in vortex shedding from each cylinder．It is noticed that the vortex－shedding frequency of MC is slightly higher than that of SC．MP pattern is similar to＇VPE＇（Vortex Pairing and Enveloping）or＇VPSE＇（Vortex Pairing， Splitting and Enveloping）patterns in Sumner et al．［5］for
$850 \leq R e \leq 1900$ ．
Synchronized Vortex Shedding pattern is noticed in the arrangements where the two cylinders is far apart（ $\mathrm{R} / \mathrm{D} \geq 2.5$ ）and $\theta \geq 20^{\circ}$ ．Due to the long distance and high inclination angle，the interaction of the wakes is negligible，and vortices are shed just like the single cylinder in the freestream．

## 3． 2 Flow－pattern Diagram



Figure 2．Flow－pattern diagram at $R e=100$ ．
Collecting all the computed results，we propose the flow－pattern diagram in the first quadrant of physical space with the main cylinder fixed at the origin（Fig．2）．Due to the flow symmetry implied in the flow configuration，the proposed diagrams still can be used，even though the surrounding cylinder is located in one of the other quadrants．The fact that the diagram（＂map＂）is useful in predicting flow patterns past two nearby circular cylinders sheds light on developing a wake－interaction model in engineering flows．

## 4．Acknowledgement

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