[50th Anniversary Symposium]

The Emergence of Collective Modes, Ecological Collapse and Directed Percolation at the Laminar-Turbulence Transition in Pipe Flow

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How do fluids become turbulent as their flow velocity is increased? In recent years, careful experiments in pipes and Taylor-Couette systems have revealed that the lifetime of transient turbulent regions in a fluid appears to diverge with flow velocity just before the onset of turbulence, faster than any power law or exponential function. I show how this superexponential scaling of the turbulent lifetime in pipe flow is reminiscent of extreme value statistics, and a manifestation of a mapping between transitional turbulence and the statistical mechanics model of directed percolation. This mapping itself arises from a further surprising and remarkable connection: laminar and turbulent regions in a fluid behave as a predator-prey ecosystem. I explain the evidence for this mapping, and propose how a unified picture of the transition to turbulence emerges in systems ranging from turbulent convection to magnetohydrodynamics.

1 Introduction

The scientific study of pipe flow turbulence was initiated by Reynolds in 1883. At that time he noted a unique feature of the laminar-turbulent transition: it occurs through the appearance of flashes of turbulence, interspersed with regions of laminar flow. The flashes are localized regions of turbulence, now known as “puffs”. Not only are they localized in space, but they are localized in time also: they have a finite non-zero lifetime as the laminar-turbulent transition is approached from below the critical Reynolds number.

During the last 10 years or so, there has been major progress on the nature of this transition, due in part to accurate measurements of the lifetime of turbulent puffs in the range of Reynolds numbers (Re) spanning around 1800-2200. The result of these studies is that the lifetime varies super-exponentially with Reynolds number, increasing with a functional form that is well-represented by the expression exp[exp(Re)]. This finding has been related to extreme value statistics [1].

It seems, surprisingly, that there is apparently no critical Re. However, above Re ~ 2100, the puffs no longer decay, but also split into two. The time it takes for a puff to split gets shorter as the Re increases. The functional form of the splitting time also grows super-exponentially as the Re is reduced from above 2100.

The central theoretical questions that we address are: (1) What is the origin of the super-exponential scaling? (2) Is there a well-defined laminar-turbulent transition, and if so, (3) what are its characteristics?

2 Method

The novel aspect of our approach was that we considered the laminar-turbulent transition to be a non-equilibrium phase transition problem, and as a result attempted to make predictions for the universality class governing the critical behavior. We attempt to approach this problem by looking for an appropriate long-wavelength effective theory. By performing direct numerical simulations to identify the important collective modes at the onset of turbulence, we uncover unexpected spatiotemporal patterns reminiscent of ecological dynamics: a large-scale zonal flow and small-scale turbulent fluctuations interact antagonistically, essentially like predator and prey in an ecosystem. This finding allowed us to write down the simplest minimal stochastic model to account for these observations, a Landau theory for the laminar-turbulent transition. This approach is a precise parallel to that used in the conventional theory of phase transitions, where one builds a Landau theory, a coarse-grained or effective theory from symmetry grounds, as indicated in Fig. 1. This model predicts, without using the Navier-Stokes equations, the puff lifetime and splitting behavior observed in experiment, as shown in Fig. 2.

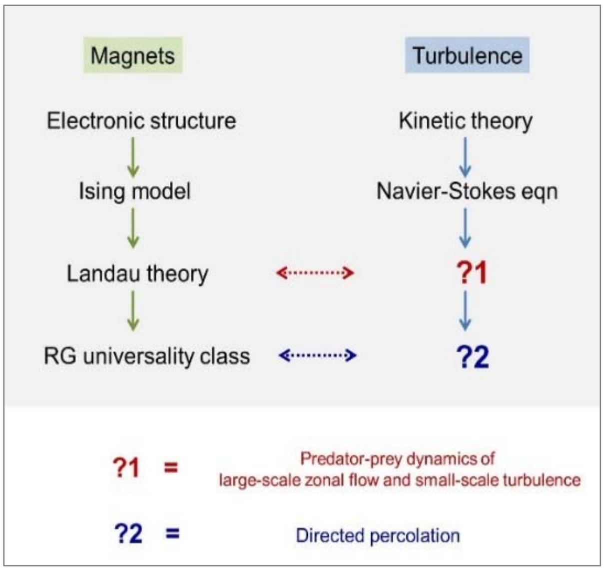

Fig. 1 Summary of the logic of modeling the laminar-turbulent phase transition, compared with steps for the conventional ferromagnetic transition of an Ising model.

3 Results and Discussion

This universality class of the laminar-turbulent transition is determined by the interplay between the small-scale turbulence and large-scale collective zonal flow that exhibit predator-prey behavior. This predator-prey model can be mapped into Reggeon field theory and thence to directed percolation (DP). DP does indeed recapitulate the experimental results[2]. Thus, the theory predicts that the dynamical interplay between zonal flow and turbulence leads to DP critical phenomena[3], a result that is in agreement with recent accurate experiments in Couette geometry. The ubiquity of predator-prey behavior is discussed, including recent studies on various transitional turbulence systems with different dimensionalities and flows. Why, then, is asymptotically divergent behavior not observed in the pipe flow experiments? The last part of this talk uses DP and a stochastic individual level model of predator-prey dynamics to investigate the relation between extreme value statistics and power law critical behavior, and shows that the paradox is resolved by considering correlated fluctuations near criticality. Surprisingly, critical fluctuations obey a universal finite-size scaling distribution whose asymptotics happen to be close to the standard extreme value distribution from which the unusual super-exponential form of turbulence decay rate can be derived [1]. To this end, it seems that the super-exponential behavior of the puff decay/splitting times as a function of Reynolds number now has an explanation, consistent with the mapping between transitional turbulence, predator-prey dynamics and DP.

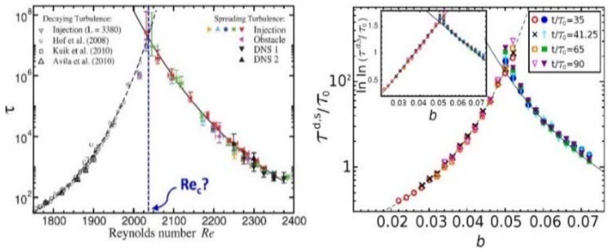

Fig. 2 Universal scaling of characteristic time scales near the laminar-turbulent transition in the pipe flow (left) and the scaling near predator extinction in the predator-prey model (right). The mean decay and splitting times of the turbulent density and the prey density scale with Reynolds number and prey birth rate with super-exponential forms.

4 Concluding Remarks

It is truly remarkable that a continuum fluid system can be accurately described by a lattice model from non-equilibrium statistical mechanics. These results and others not presented here indicate that the turbulence problem can be usefully thought of using statistical mechanics[4].

Acknowledgments: This work was partly supported by the United States National Science Foundation through grant NSF-DMR-1044901.

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