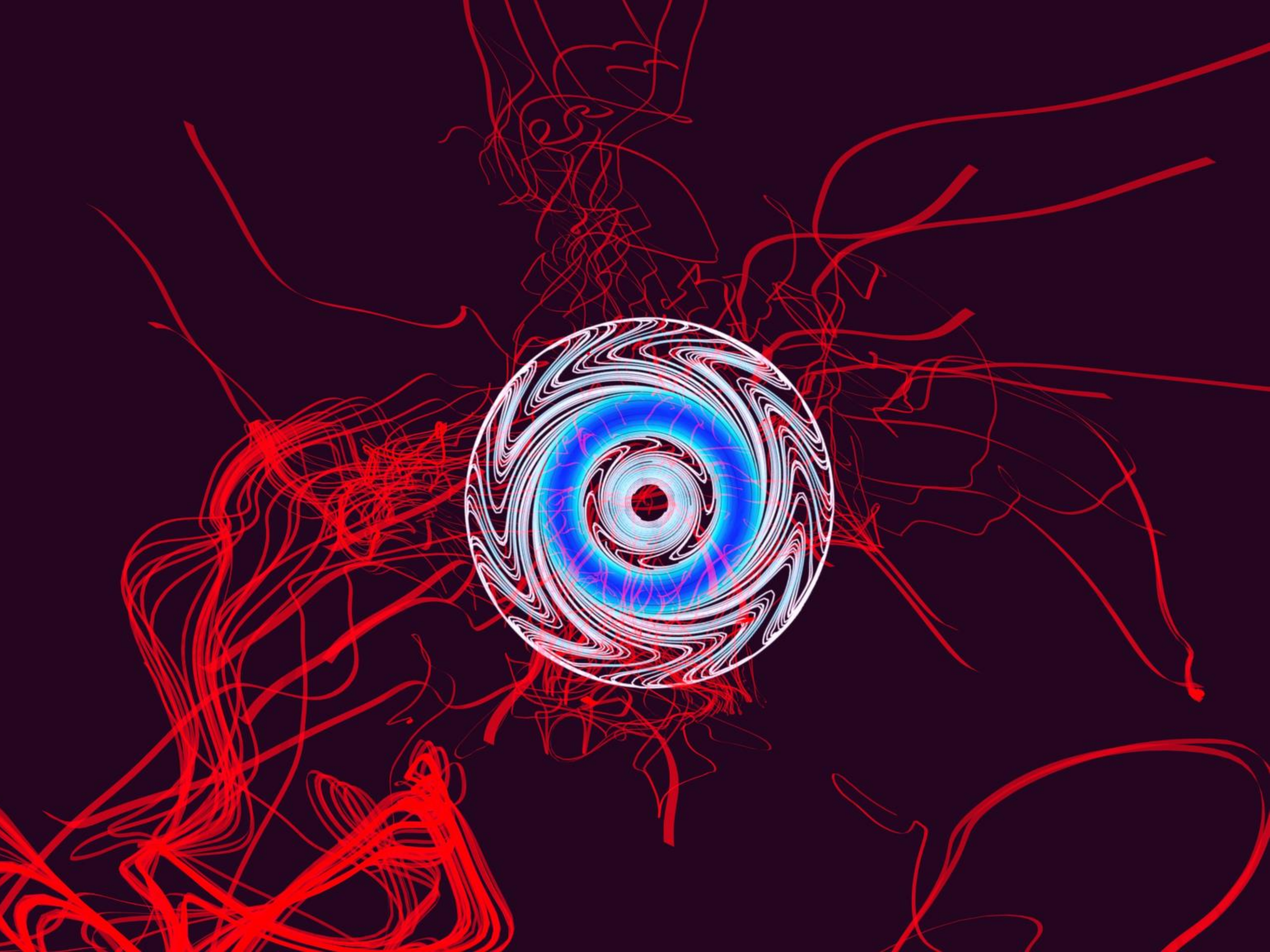
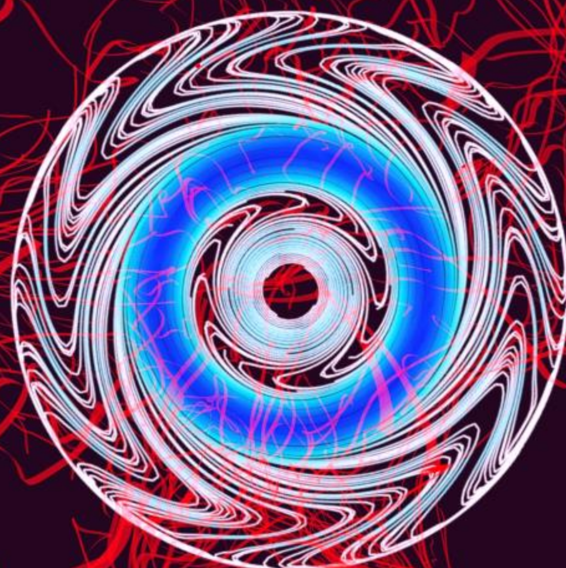
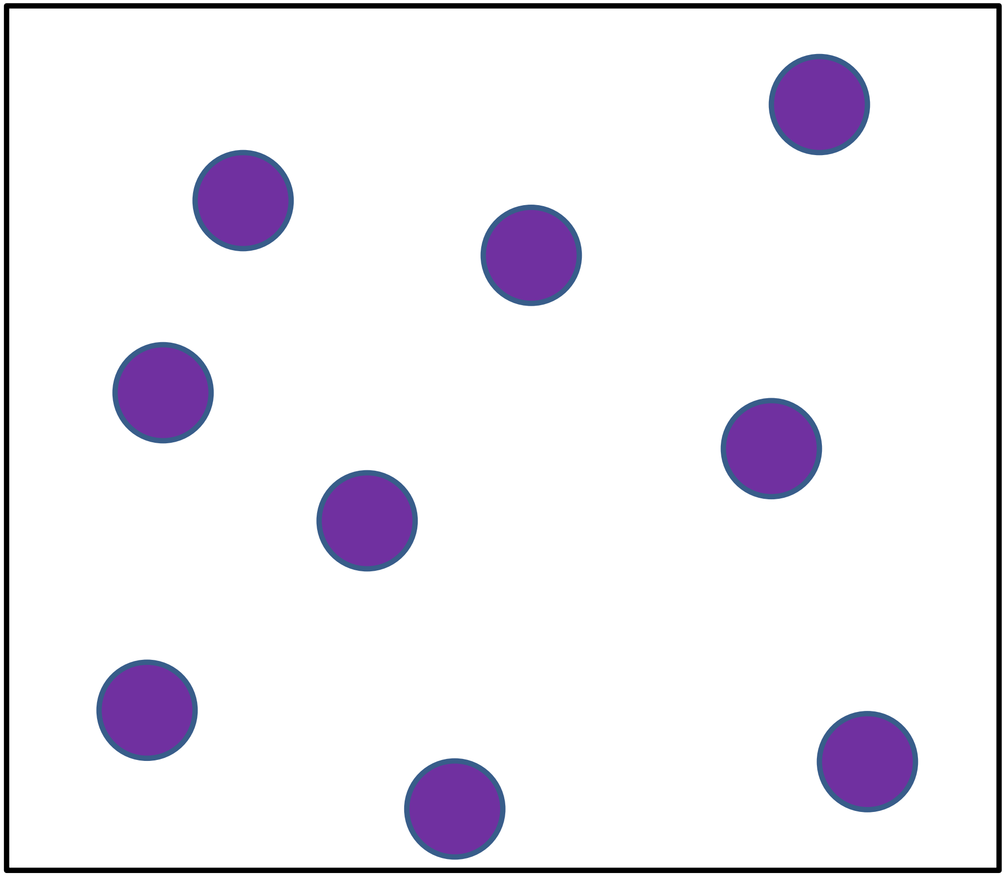


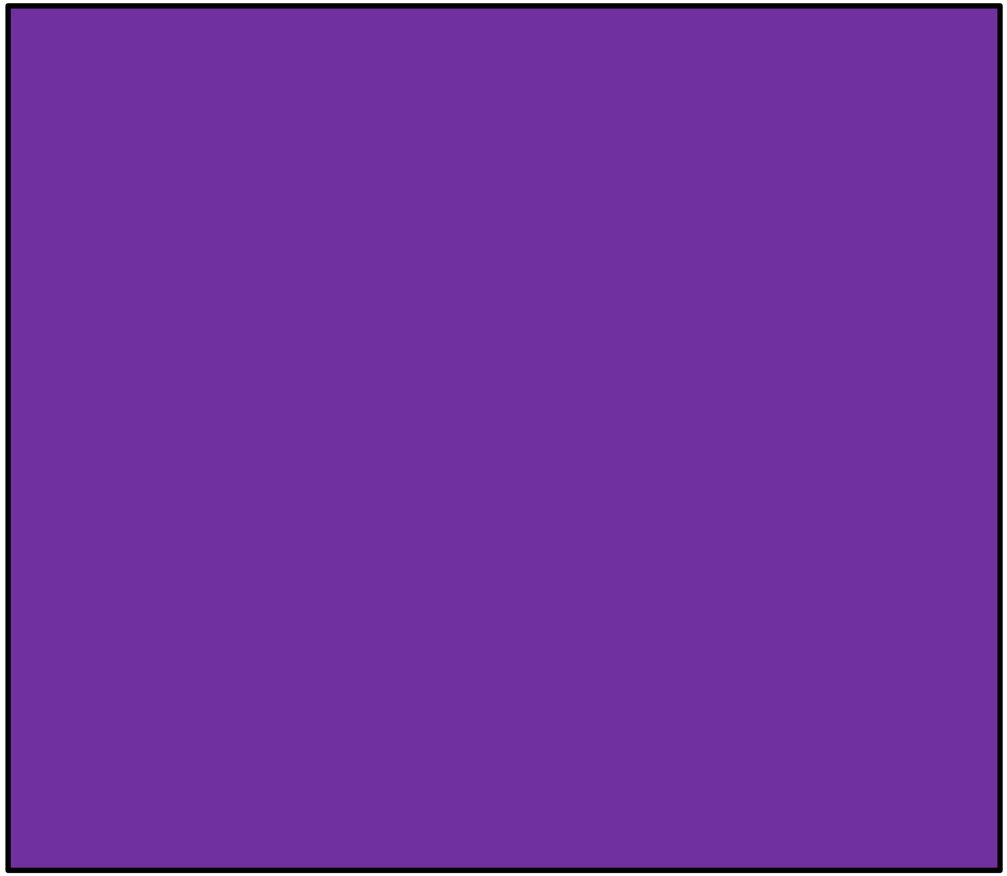
Emergence of collective modes, ecological collapse and directed percolation at the laminar-turbulence transition in pipe flow



Hong-Yan Shih, Tsung-Lin Hsieh, Nigel Goldenfeld
University of Illinois at Urbana-Champaign
Partially supported by NSF-DMR-1044901
H.-Y. Shih, T.-L. Hsieh and N. Goldenfeld, *Nature Physics* **12**, 245 (2016)
N. Goldenfeld and H.-Y. Shih, *J. Stat. Phys.* **167**, 575-594 (2017)



Deterministic classical mechanics of many particles in a box ➔ statistical mechanics



Deterministic classical mechanics of infinite number of particles in a box

= Navier-Stokes equations for a fluid

➔ statistical mechanics

$$\nabla \cdot \mathbf{u} = 0$$

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = -\nabla p + \mu\nabla^2\mathbf{u}$$

Deterministic classical mechanics of infinite number of particles in a box

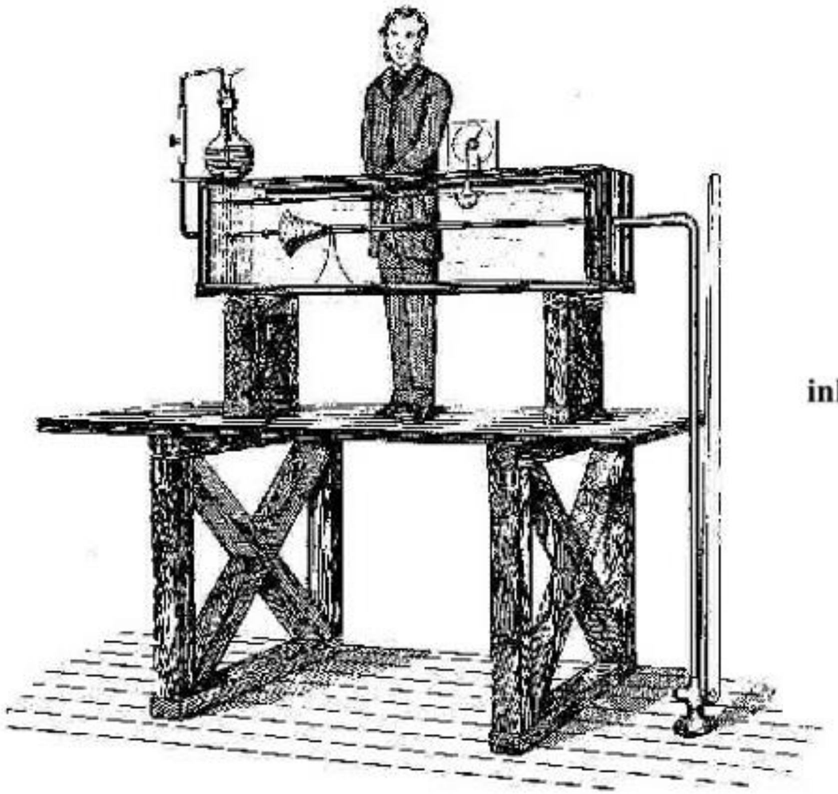
= Navier-Stokes equations for a fluid

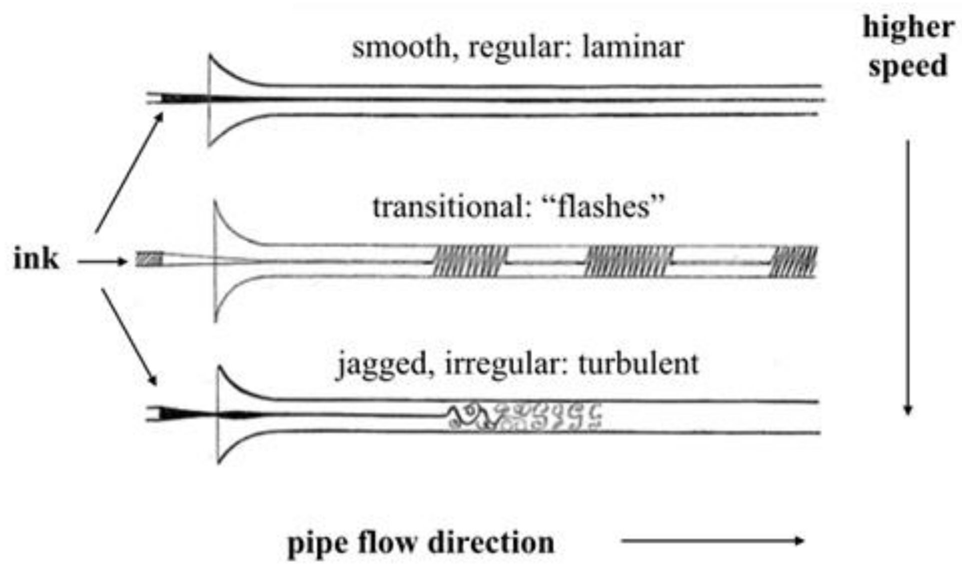
➔ statistical mechanics

Transitional turbulence: puffs

- **Reynolds' original pipe turbulence (1883) reports on the transition**

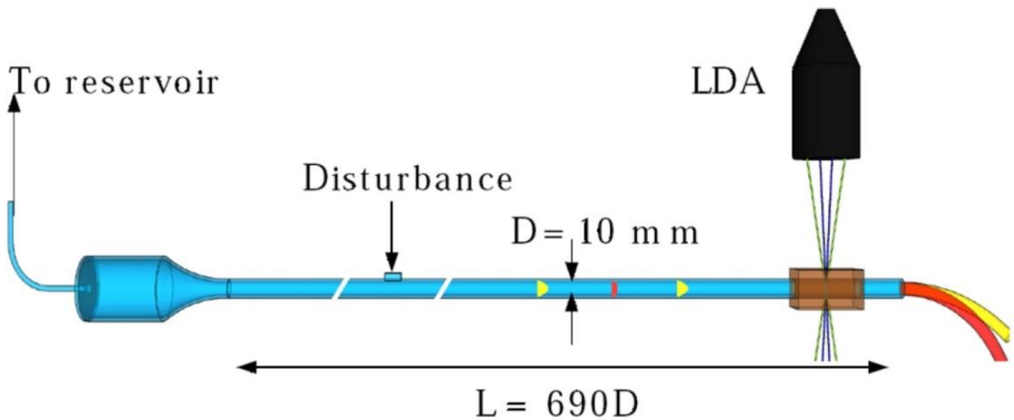
Univ. of Manchester





Precision measurement of turbulent transition

Q: will a puff survive to the end of the pipe?



Many repetitions ➔ survival probability = P(Re, t)

Hof et al., *PRL* **101**, 214501 (2008)

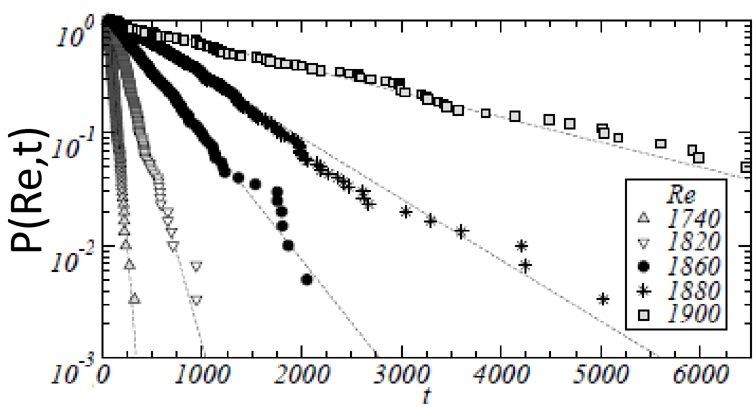
Pipe flow turbulence

Decaying single puff

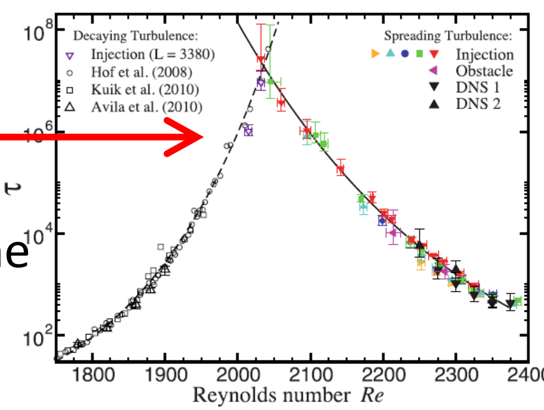
laminar	metastable puffs	spatiotemporal intermittency	expanding slugs

Re 1775 2050 2500

Survival probability $P(\mathrm{Re}, t) = e^{-\frac{t-t_0}{\tau(\mathrm{Re})}}$

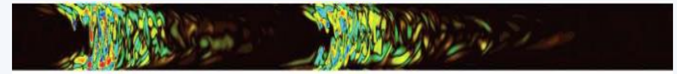


Puff lifetime



Avila *et al.*, (2009) Avila *et al.*, *Science* **333**, 192 (2011) Hof *et al.*, *PRL* **101**, 214501 (2008)

Pipe flow turbulence



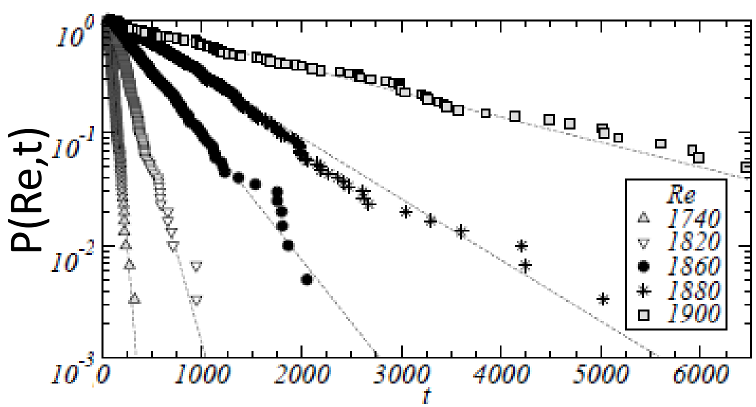
Decaying single puff

Splitting puffs

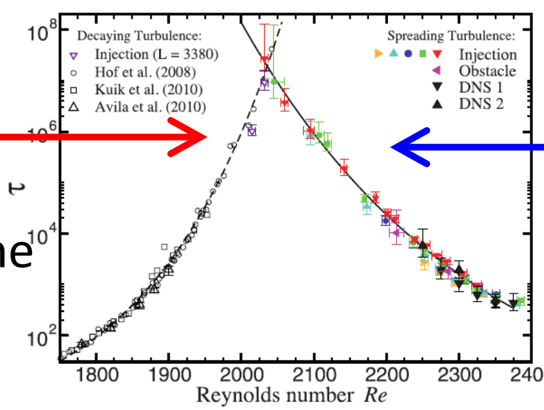
laminar | metastable puffs | spatiotemporal intermittency | expanding slugs

Re 1775 2050 2500

Splitting probability $1 - P(\mathrm{Re}, t) = e^{-\frac{t - t_0}{\tau(\mathrm{Re})}}$



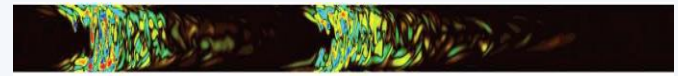
Puff lifetime



Mean time between split events

Avila *et al.*, (2009) Avila *et al.*, *Science* **333**, 192 (2011) Hof *et al.*, *PRL* **101**, 214501 (2008)

Pipe flow turbulence



Decaying single puff

Splitting puffs

laminar	metastable puffs	spatiotemporal intermittency	expanding slugs

Re — 1775 — 2050 — 2500

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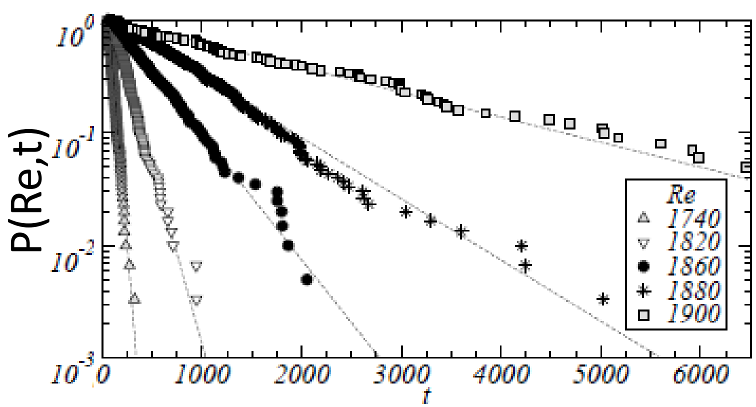
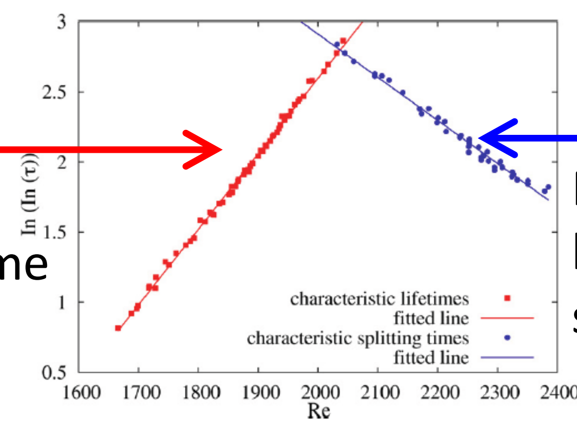


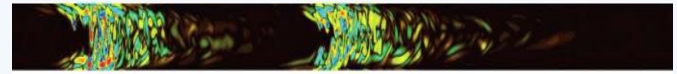
Puff lifetime



Mean time between split events

Avila *et al.*, (2009) Avila *et al.*, *Science* **333**, 192 (2011) Hof *et al.*, *PRL* **101**, 214501 (2008)

Pipe flow turbulence



Decaying single puff

Splitting puffs

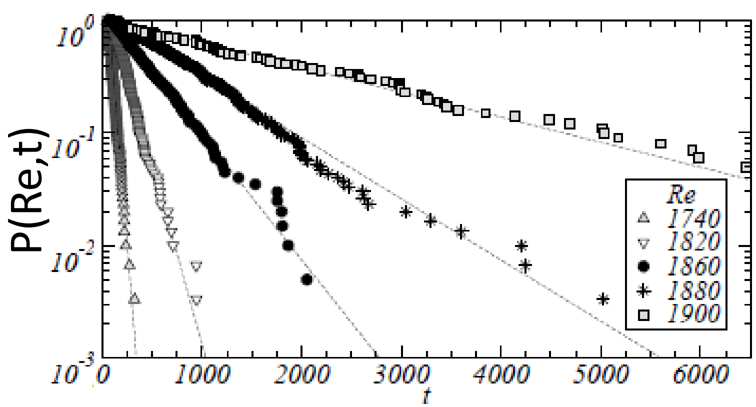
laminar | metastable puffs | spatiotemporal intermittency | expanding slugs

Re

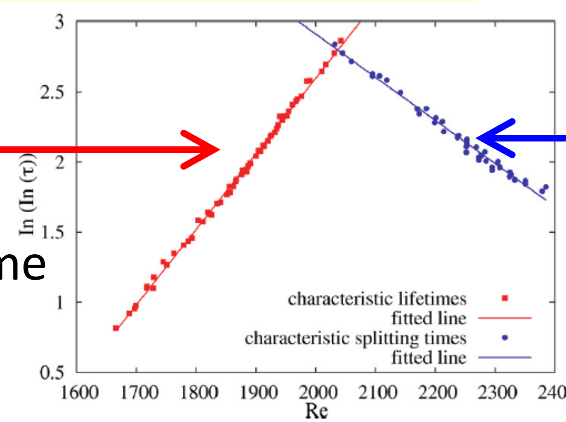
Super-exponential scaling:

$$\frac{\tau}{\tau_0} \sim \exp(\exp \mathrm{Re})$$

Survival p



Puff lifetime



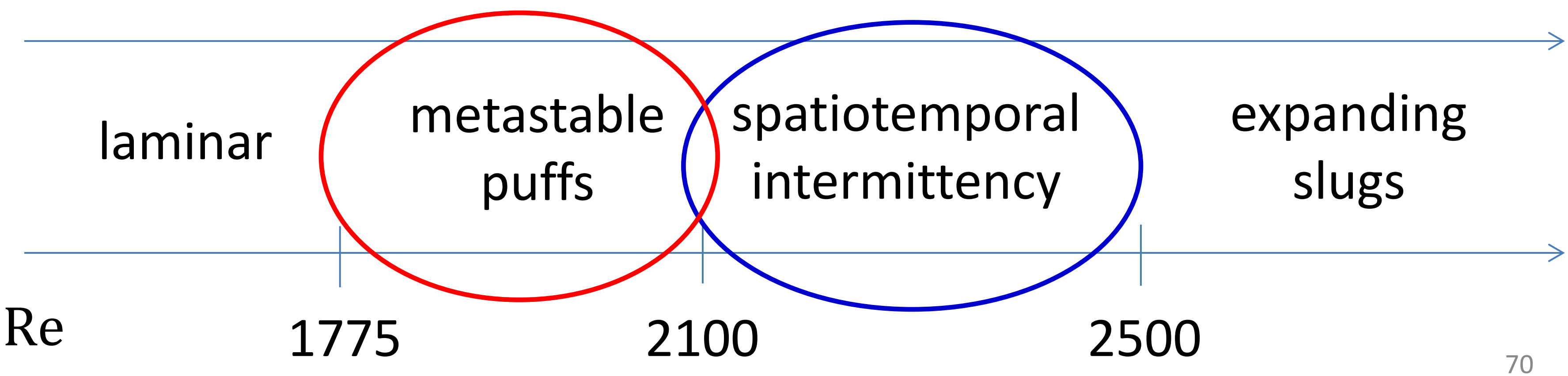
Mean time between split events

Avila *et al.*, (2009)   Avila *et al.*, *Science* **333**, 192 (2011)   Hof *et al.*, *PRL* **101**, 214501 (2008)

MODEL FOR METASTABLE TURBULENT PUFFS & SPATIOTEMPORAL INTERMITTENCY

Shih, Hsieh and Goldenfeld, Nature Physics (2016)

Very complex behavior and we need to understand precisely what happens at the transition, and where the DP universality class comes from.



Logic of modeling phase transitions

Magnets

Electronic structure

↓

Ising model

↓

Landau theory

↓

RG universality class
(Ising universality class)

Logic of modeling phase transitions

Magnets	Turbulence
Electronic structure	Kinetic theory
↓	↓
Ising model	Navier-Stokes eqn
↓	↓
Landau theory	**?**
↓	↓
RG universality class (Ising universality class)	**?**

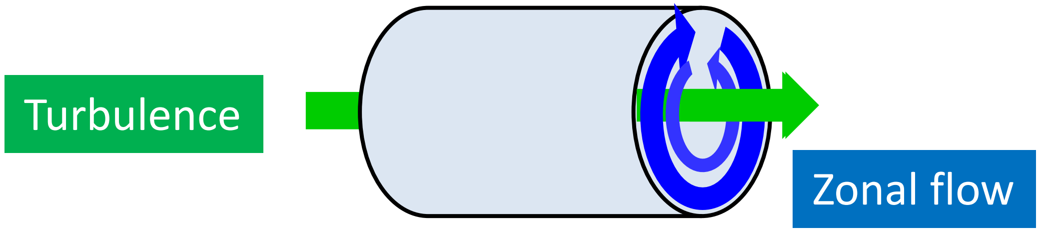
Logic of modeling phase transitions

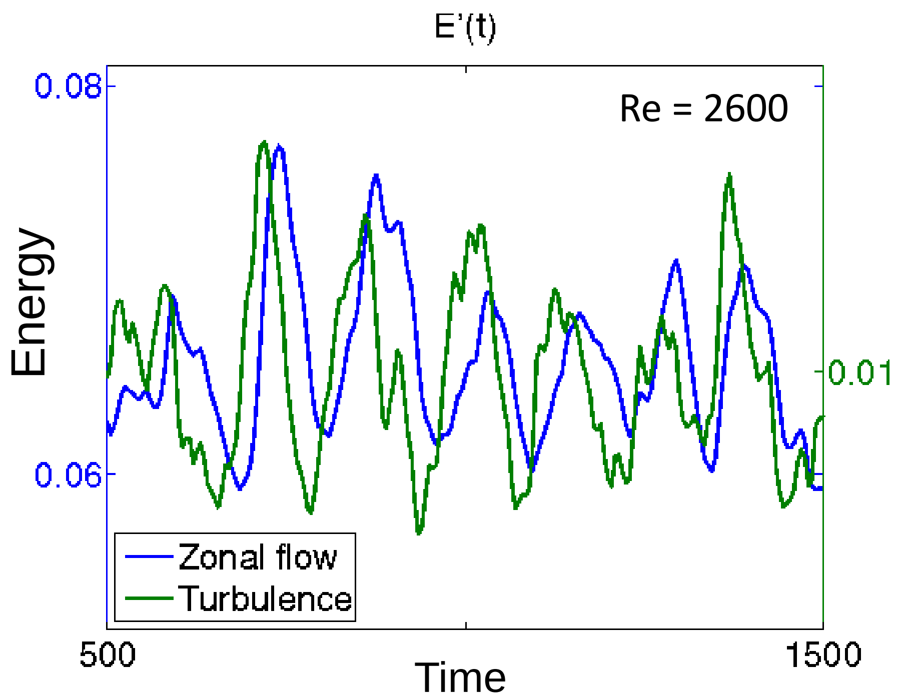
Magnets	Turbulence
Electronic structure	Kinetic theory
Ising model	Navier-Stokes eqn
Landau theory	**?**
RG universality class (Ising universality class)	**?**

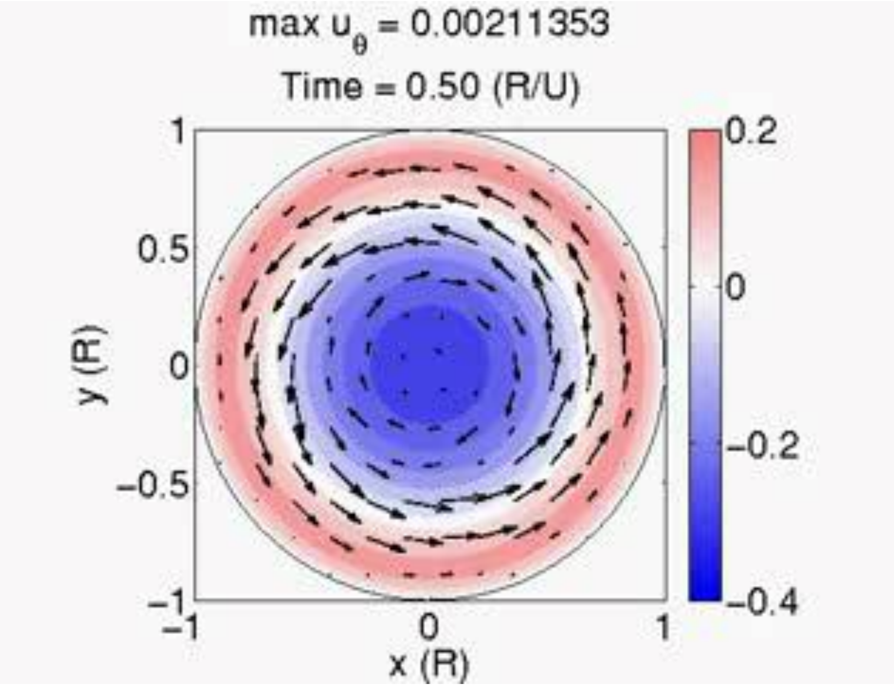
Identification of collective modes at the laminar-turbulent transition

To avoid technical approximations,
we use DNS of Navier-Stokes

Predator-prey oscillations in pipe flow



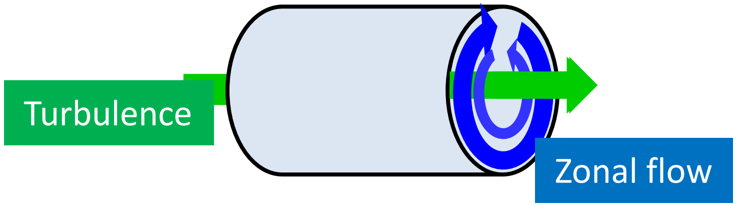




Simulation based on the open source
code by Ashley Willis: openpipeflow.org

What drives the zonal flow?

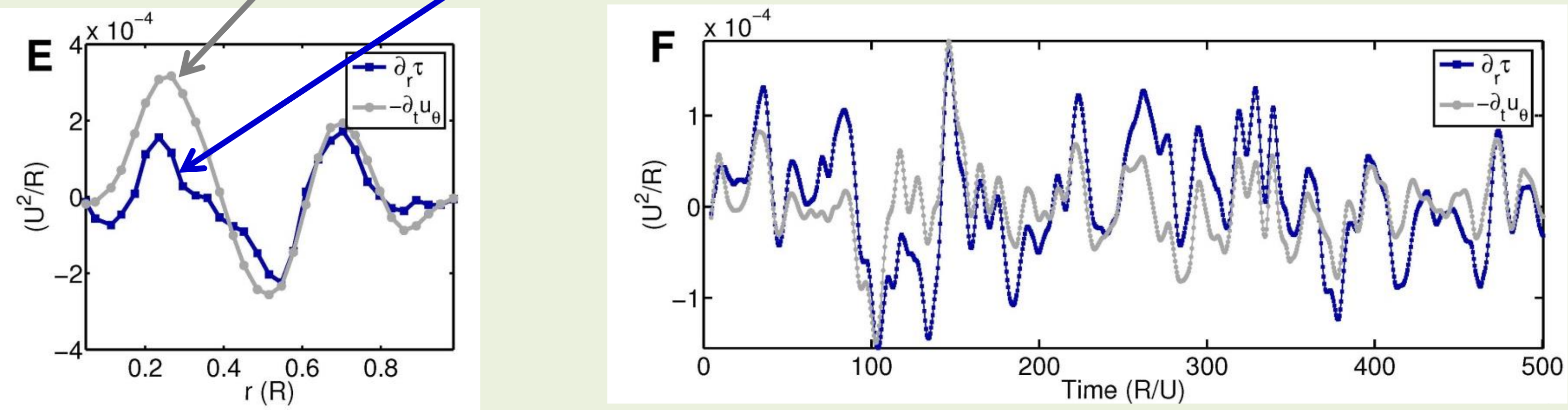
- **Interaction in two fluid model**
  - Turbulence, small-scale (k>0)
  - Zonal flow, large-scale (k=0,m=0): induced by turbulence and creates shear to suppress turbulence



1) Anisotropy of turbulence creates Reynolds stress which generates the mean velocity in azimuthal direction

$$\partial_t \langle v_\theta \rangle = -\partial_r \langle (\widetilde{v}_\theta \cdot \widetilde{v}_r) \rangle - \mu \langle v_\theta \rangle$$

2) Mean azimuthal velocity decreases the anisotropy of turbulence and thus suppress turbulence



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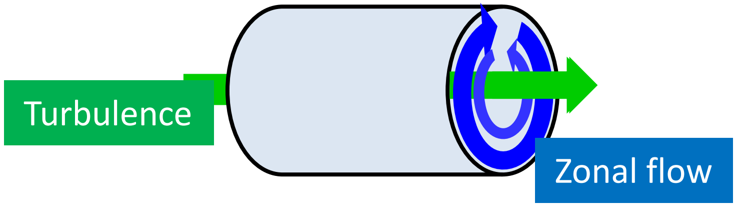

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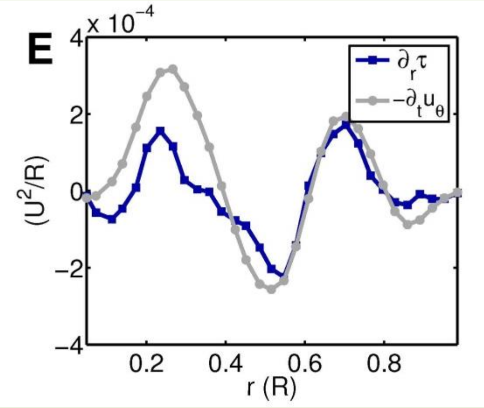

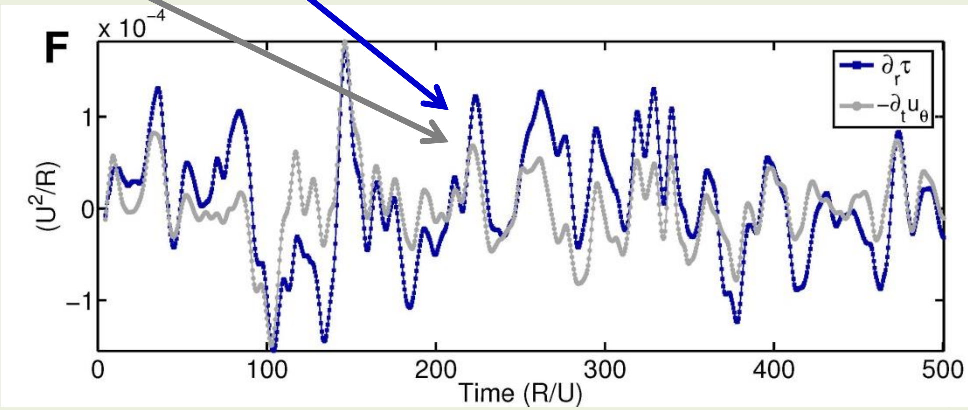

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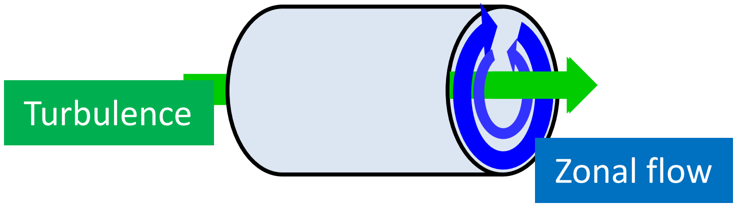

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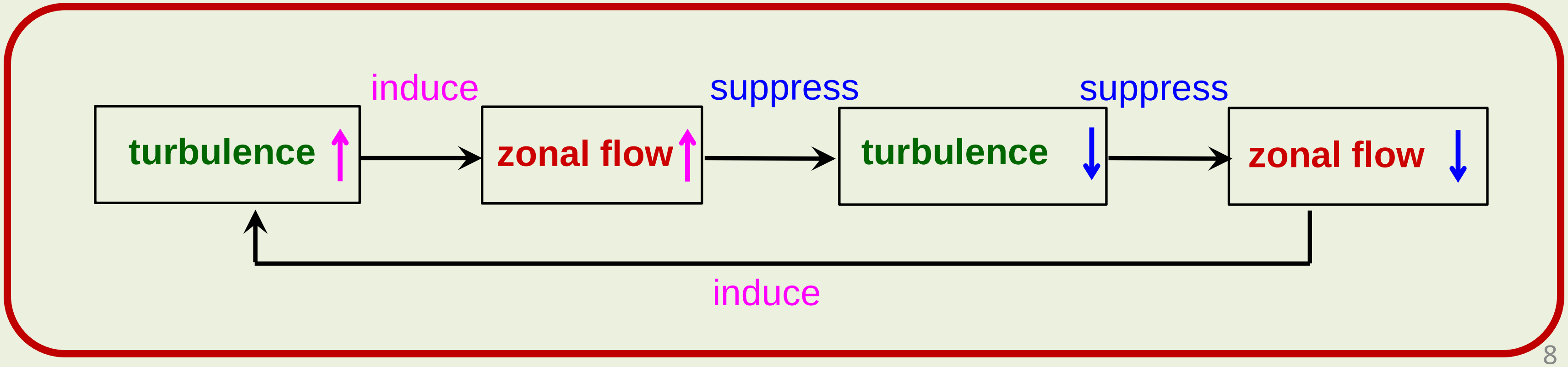

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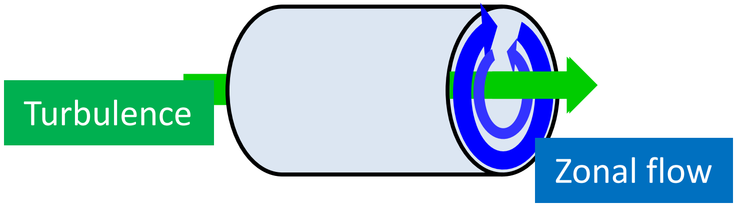

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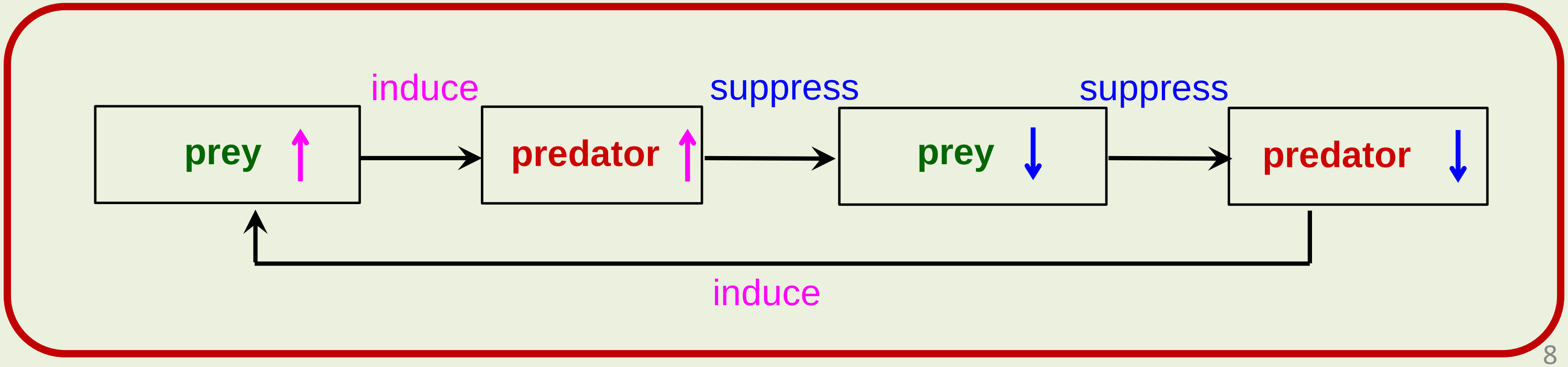
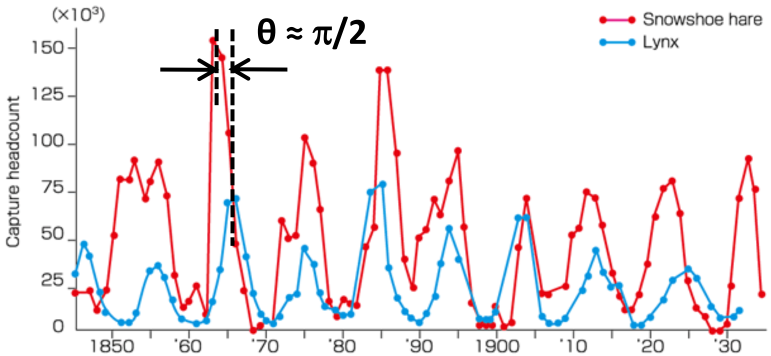


Population cycles in a predator-prey system

$\pi/2$ phase shift between prey and predator population



Persistent oscillations
+
Fluctuations

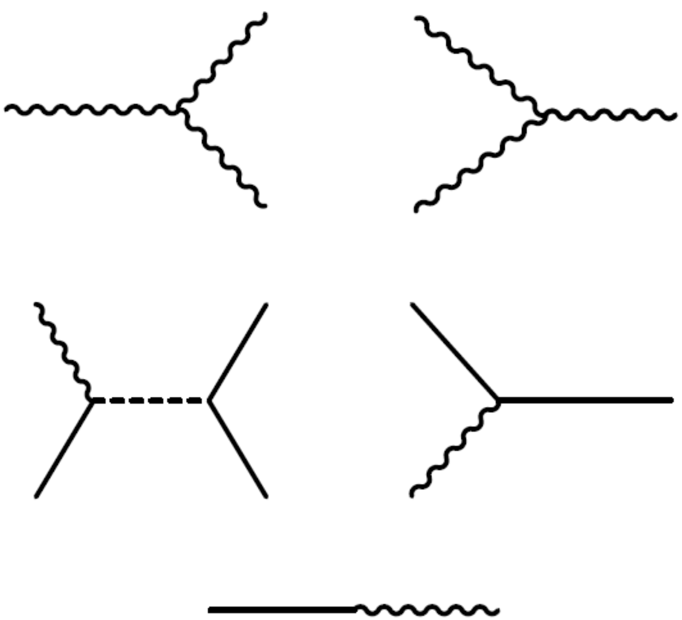
https://interstices.info/jcms/n_49876/des-especes-en-nombre

Derivation of predator-prey equations

Zonal flow Turbulence
Vacuum = Laminar flow

A = predator B = prey
E = food/empty state

Zonal flow-turbulence



Predator-prey

$$B + E \xrightarrow{b} B + B$$

$$B + B \xrightarrow{c} B + E$$

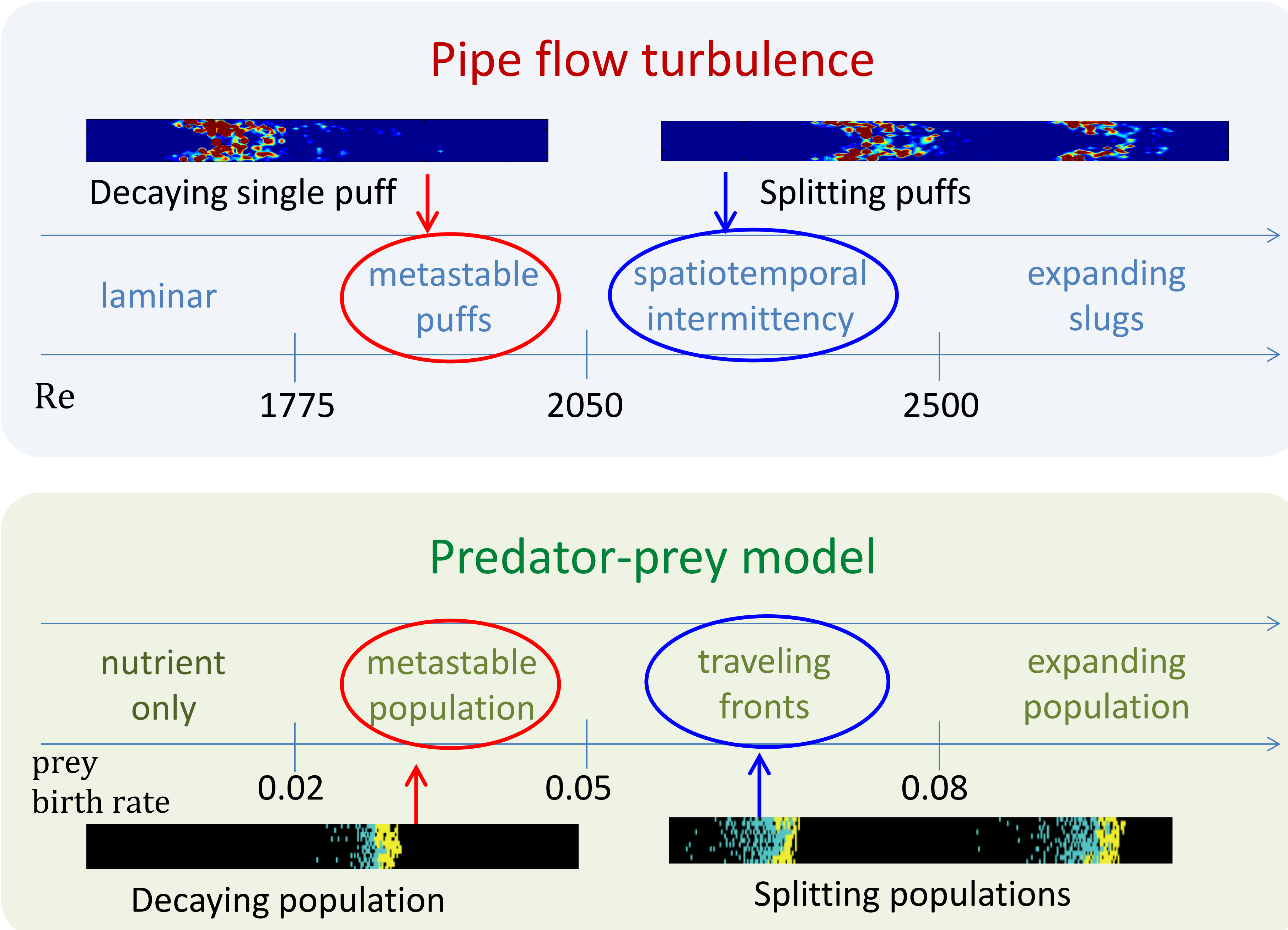
$$A + B \xrightarrow{p} A + A$$

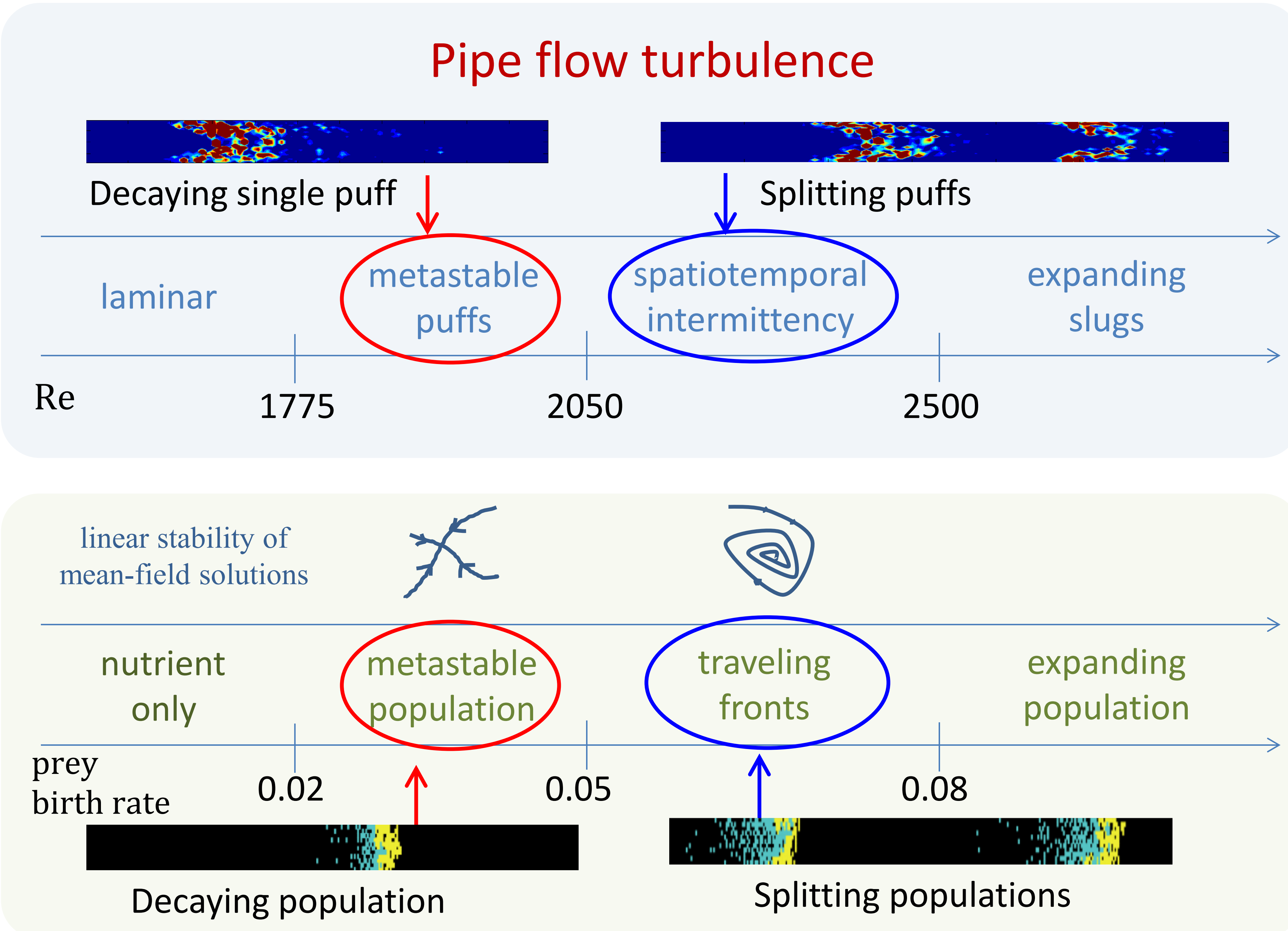
$$A + B \xrightarrow{p'} A + E$$

$$B \xrightarrow{m} A$$

$$A \xrightarrow{d_A} E \quad B \xrightarrow{d_B} E$$

Extinction/decay statistics for stochastic predator-prey systems


Pipe flow turbulence
Decaying single puff
Splitting puffs
laminar
metastable puffs
spatiotemporal intermittency
expanding slugs
Re
1775
2050
2500
Predator-prey model
nutrient only
metastable population
traveling fronts
expanding population
prey birth rate
0.02
0.05
0.08
Decaying population
Splitting populations



Pipe flow turbulence
Decaying single puff
Splitting puffs
laminar
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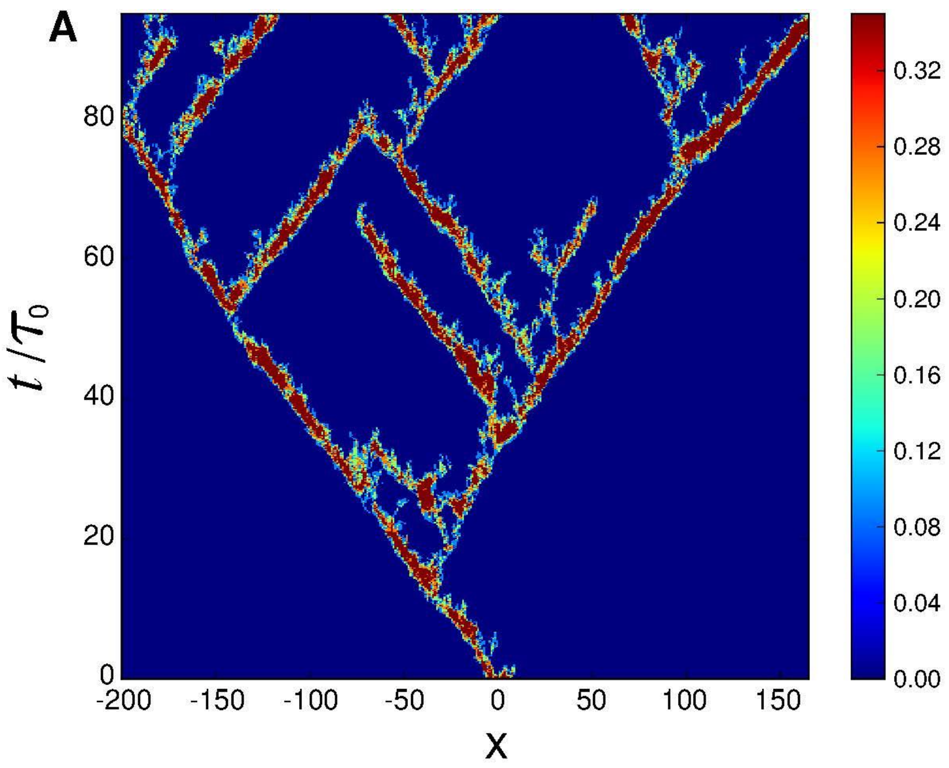
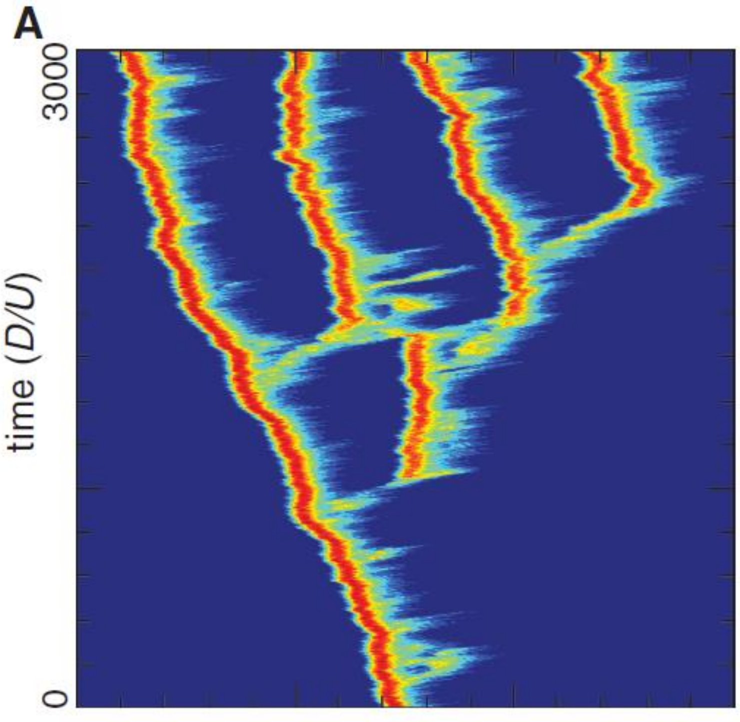
spatiotemporal intermittency
expanding slugs
Re
1775
2050
2500
linear stability of mean-field solutions
nutrient only
metastable population
traveling fronts
expanding population
prey birth rate
0.02
0.05
0.08
Decaying population
Splitting populations

Puff splitting in predator-prey systems



Puff-splitting in predator-prey ecosystem in a pipe geometry



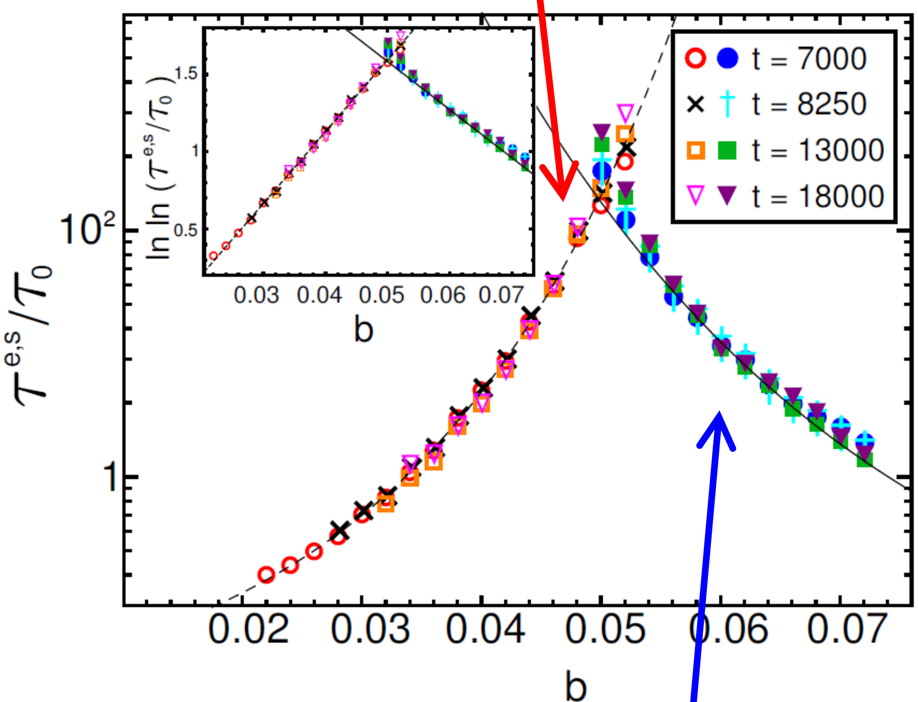
Puff-splitting in pipe turbulence

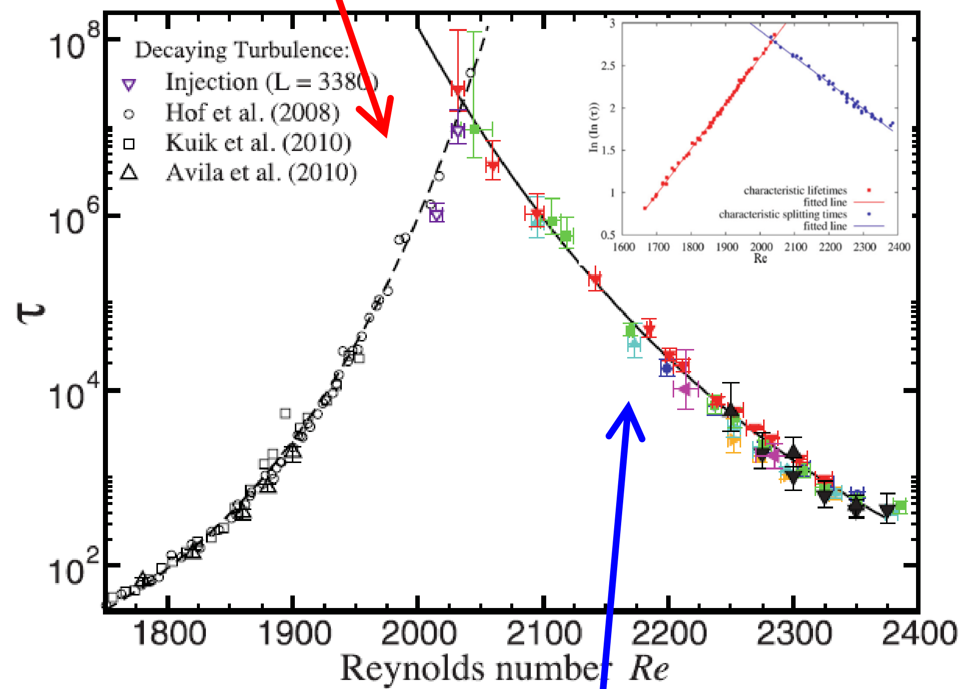
Avila et al., Science (2011)

Predator-prey vs. transitional turbulence

Prey lifetime

Turbulent puff lifetime





Mean time between population split events

Mean time between puff split events

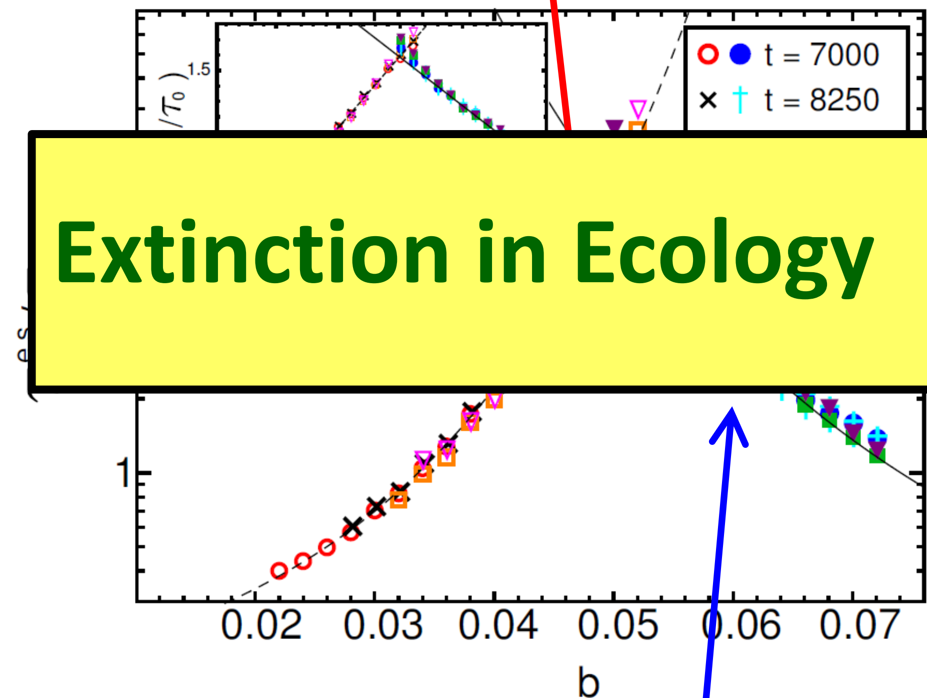
Avila *et al.*, *Science* **333**, 192 (2011)

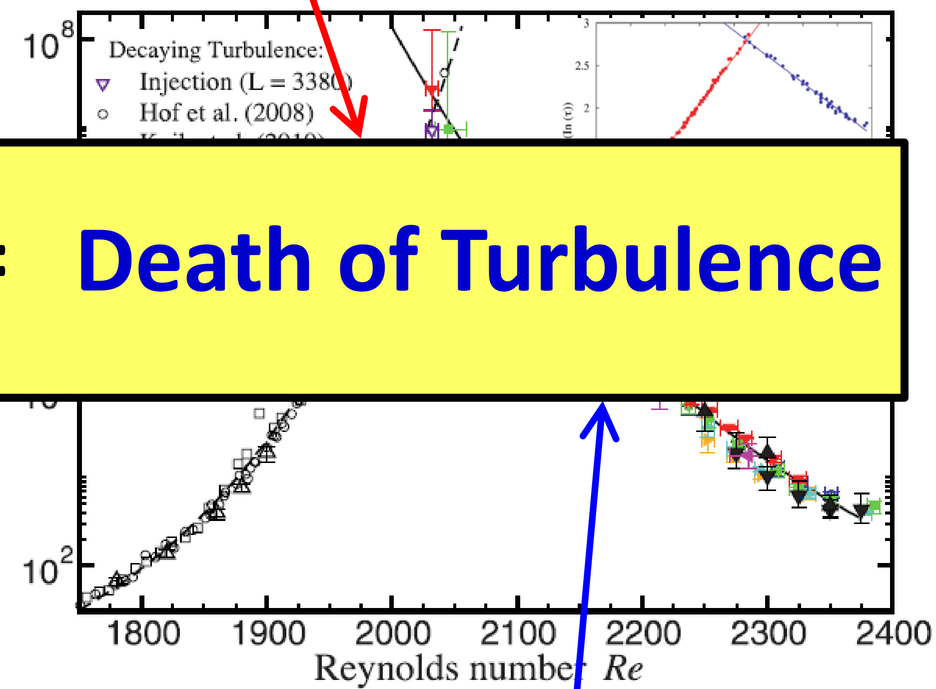
Song *et al.*, *J. Stat. Mech.* 2014(2), P020010

Predator-prey vs. transitional turbulence

Prey lifetime

Turbulent puff lifetime





Extinction in Ecology = Death of Turbulence

Mean time between population split events

Mean time between puff split events

Avila *et al.*, *Science* **333**, 192 (2011)

Song *et al.*, *J. Stat. Mech.* 2014(2**)**, P020010

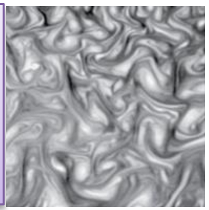
Roadmap: Universality class of laminar-turbulent transition

Universality class

?

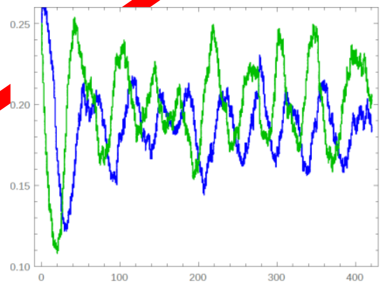
(Boffetta and Ecke, 2012)

(Classical) Turbulence



Direct Numerical Simulations of Navier-Stokes

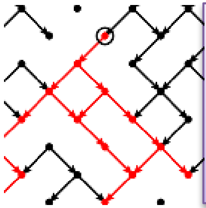
Two-fluid model



Predator-Prey

(Pearson Education, Inc., 2009)

Roadmap: Universality class of laminar-turbulent transition



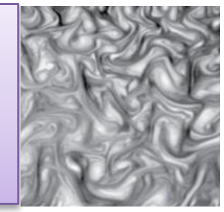
Directed Percolation

(Wikimedia Commons)

?

(Boffetta and Ecke, 2012)

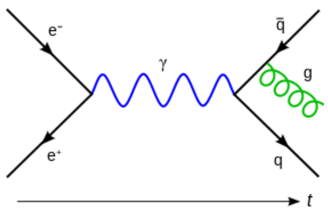
(Classical) Turbulence



Reggeon field theory
(Janssen, 1981)

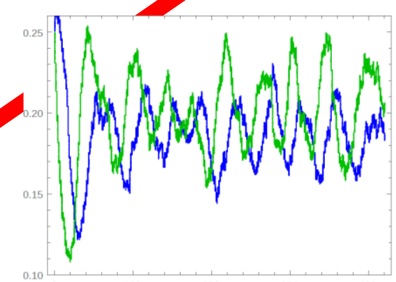
Direct Numerical Simulations
of Navier-Stokes

Field Theory



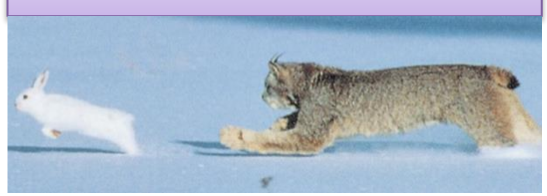
(Wikimedia Commons)

Two-fluid model



Extinction transition
(Mobilia et al., 2007)

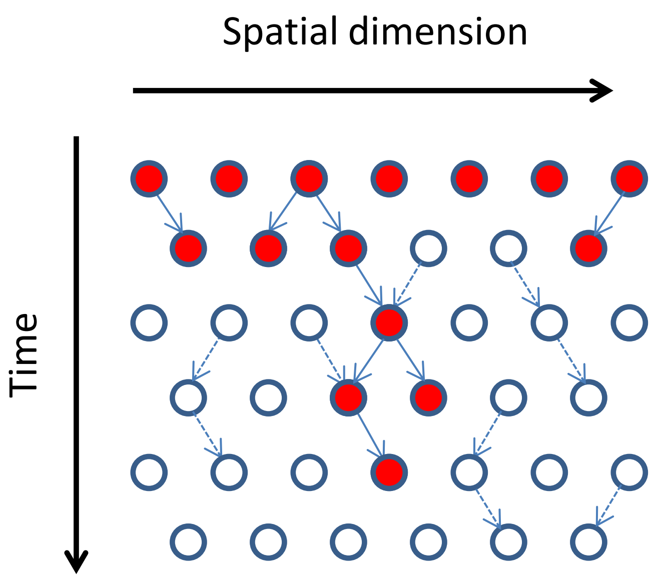
Predator-Prey

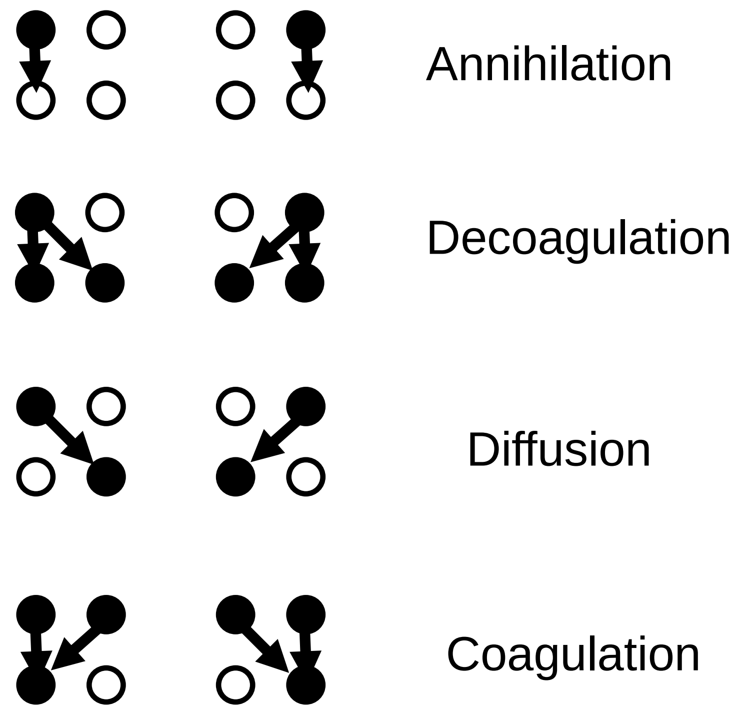


(Pearson Education, Inc., 2009)

Directed percolation & the laminar-turbulent transition

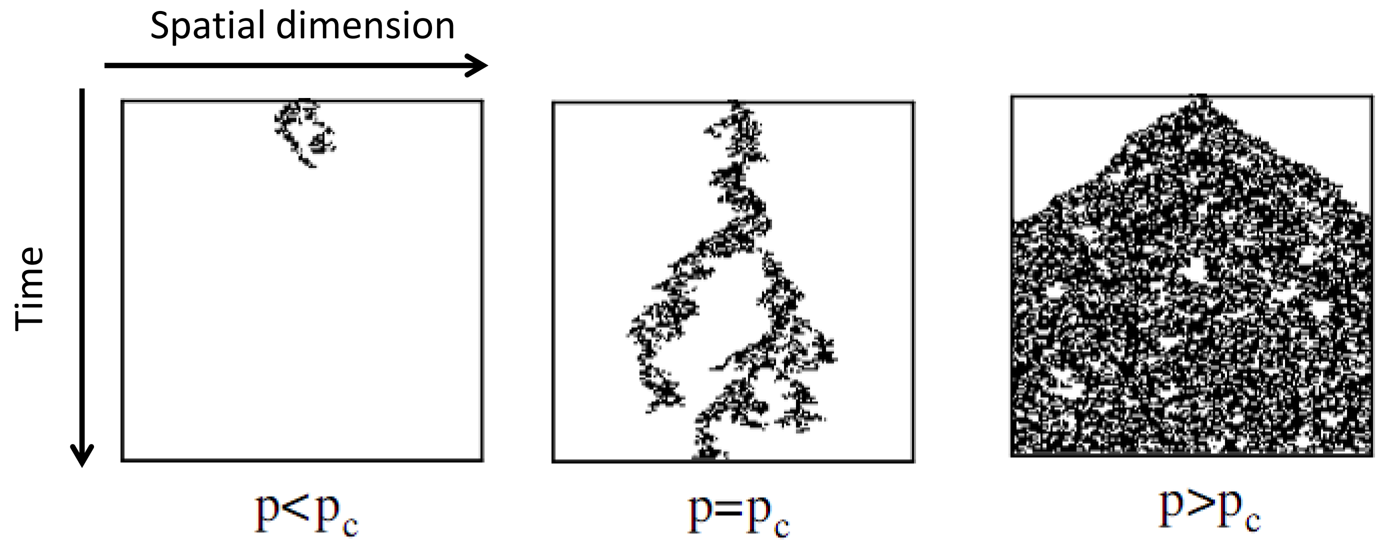
- Turbulent regions can spontaneously relaminarize (go into an absorbing state).
- They can also contaminate their neighbourhood with turbulence. (Pomeau 1986)





Directed percolation transition

- A continuous phase transition occurs at p_c.



Hinrichsen (Adv. in Physics 2000)

- Phase transition characterized by universal exponents:

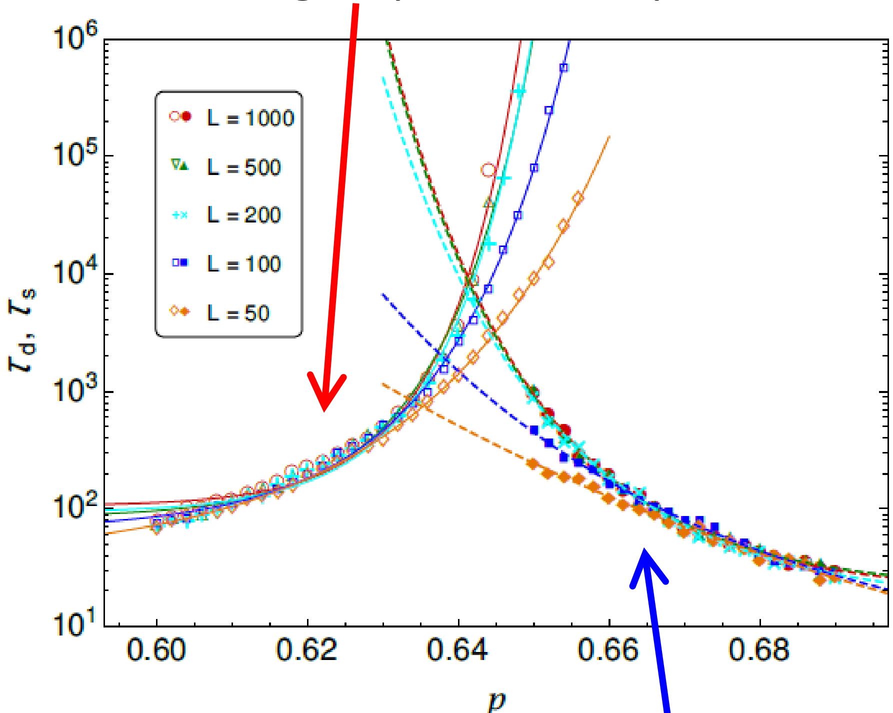
$$\rho \sim (p - p_c)^{\beta} \qquad \xi_{\perp} \sim (p - p_c)^{-\nu_{\perp}} \qquad \xi_{\parallel} \sim (p - p_c)^{-\nu_{\parallel}}$$

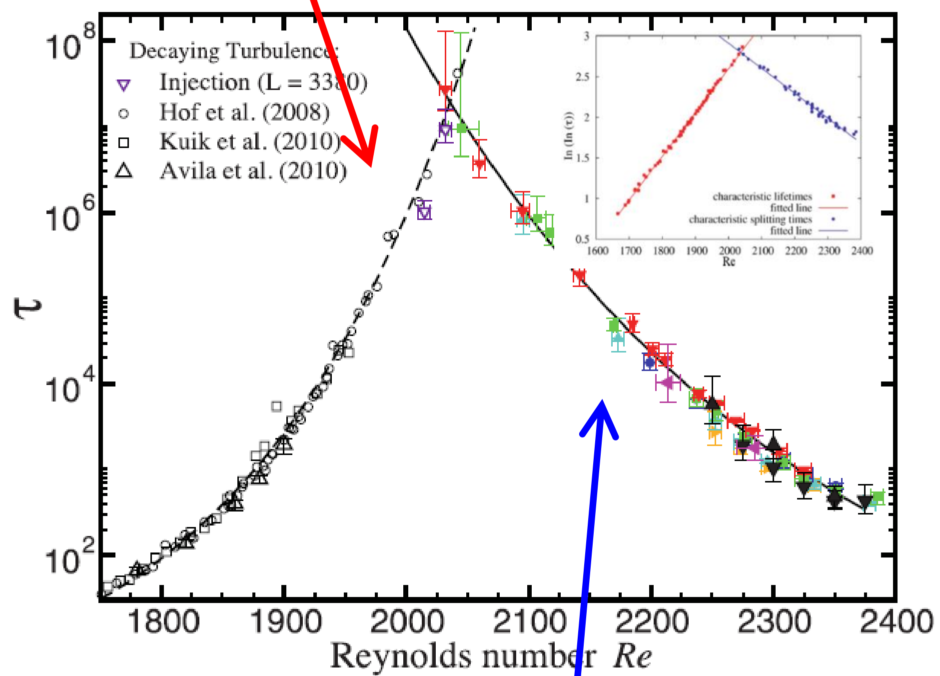
Directed percolation vs. transitional turbulence

Survival probability $P(\mathrm{Re}, t) = e^{-\frac{t-t_0}{\tau(\mathrm{Re})}}$

Longest percolation path

Turbulent puff lifetime





Longest length of empty site

Mean time between puff split events

Sipos and Goldenfeld (2011) Shih and Goldenfeld (in preparation) Avila *et al.*, (2011) Song *et al.*, (2014)

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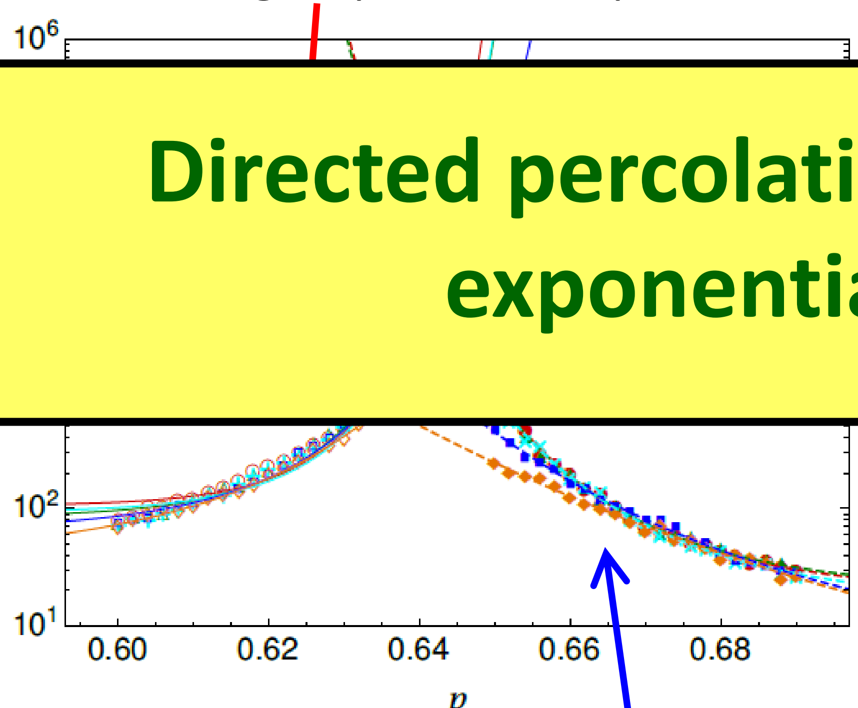

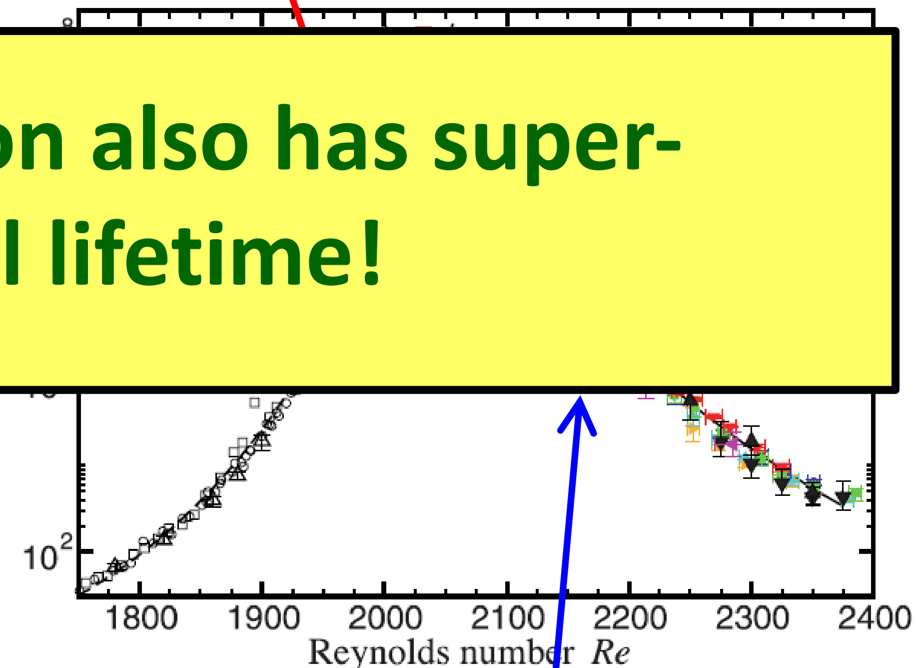

Directed percolation also has super-exponential lifetime!

Longest length of empty site

Mean time between puff split events

Sipos and Goldenfeld (2011) Shih and Goldenfeld (in preparation) Avila *et al.*, (2011) Song *et al.*, (2014)

Predator-prey & DP: connection?

- Near the laminar-turbulent transition, two important modes behave like predator-prey

- Near the laminar-turbulent transition, lifetime statistics grow super-exponentially with Re, behaving like directed percolation

- **How can both descriptions be valid?**

Universality class of predator-prey system near extinction

Basic individual processes in predator (A) and prey (B) system:

Death $B_i \xrightarrow{d_B} E_i$ $\quad A_i \xrightarrow{d_A} E_i$ $\quad B_i \xrightarrow{m} A_i$

Birth $B_i + E_j \xrightarrow[\langle ij \rangle]{b} B_i + B_j$ $\quad A_i + B_j \xrightarrow[\langle ij \rangle]{p} A_i + A_j$

Diffusion $B_i + E_j \xrightarrow[\langle ij \rangle]{D} E_i + B_j$ $\quad A_i + E_j \xrightarrow[\langle ij \rangle]{D} E_i + A_j$

Carrying capacity $B_i + B_j \xrightarrow[\langle ij \rangle]{c} B_i + E_j$

Universality class of predator-prey system near extinction

Near the transition to prey extinction, the prey (B) population is very small and no predator (A) can survive; A ~ 0.

Death $B_i \xrightarrow{d_B} E_i$ $\quad A_i \cancel{\xrightarrow{d_A}} E_i$ $\quad B_i \cancel{\xrightarrow{m}} A_i$

Birth $B_i + E_j \xrightarrow[\langle ij \rangle]{b} B_i + B_j$ $\quad A_i + B_j \cancel{\xrightarrow[\langle ij \rangle]{p}} A_i + A_j$

Diffusion $B_i + E_j \xrightarrow[\langle ij \rangle]{D} E_i + B_j$ $\quad A_i + E_j \cancel{\xrightarrow[\langle ij \rangle]{D}} E_i + A_j$

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Diffusion $$B_i + E_j \xrightarrow[\langle ij \rangle]{D} E_i + B_j$$

Carrying capacity $$B_i + B_j \xrightarrow[\langle ij \rangle]{c} B_i + E_j$$

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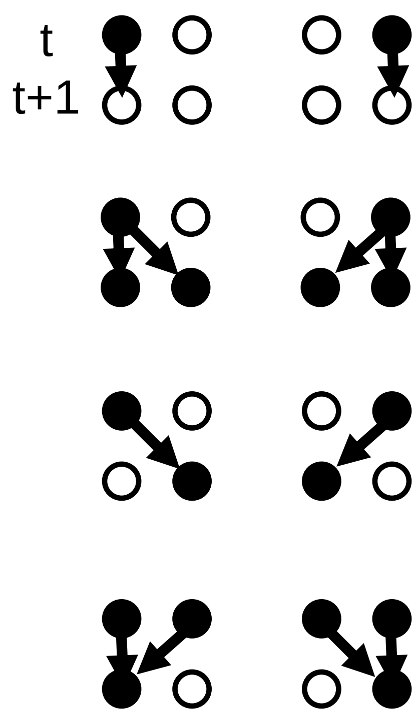
Carrying capacity $B_i + B_j \xrightarrow[\langle ij \rangle]{c} B_i + E_j$



Annihilation

Decoagulation

Diffusion

Coagulation

Universality class of predator-prey system near extinction

Near the transition to prey extinction, the prey (B)
populatio… …n
survive; …

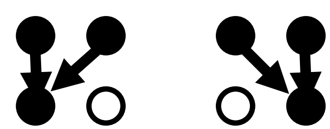
Near the extinction transition, stochastic predator-prey dynamics reduces to directed percolation

Death $B_i \xrightarrow{d_B}$ … …ation

Birth $B_i + E$ … …ulation

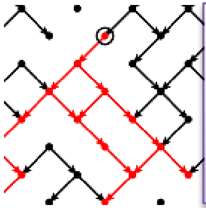
Diffusion $B_i + E_j$ … …usion

Carrying capacity $B_i + B_j \xrightarrow[\langle ij \rangle]{c} B_i + E_j$



Coagulation

Summary: universality class of transitional turbulence

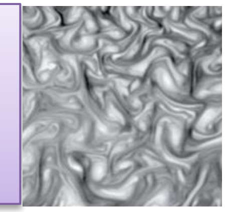


(Wikimedia Commons)

Directed Percolation

(Boffetta and Ecke, 2012)

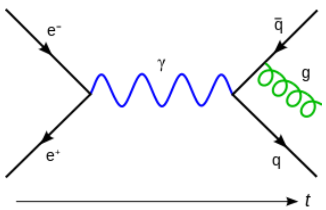
(Classical) Turbulence



Reggeon field theory (Janssen, 1981)

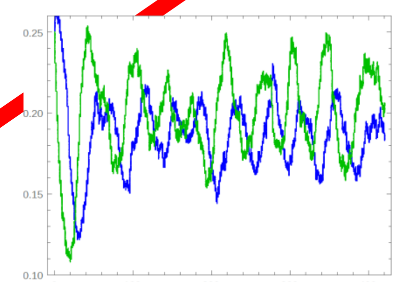
Direct Numerical Simulations of Navier-Stokes

Field Theory



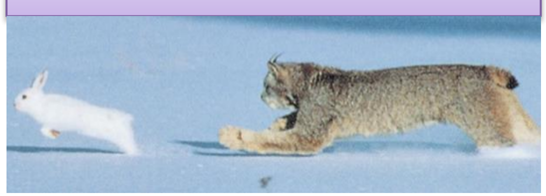
(Wikimedia Commons)

Two-fluid model



Extinction transition (Mobilia et al., 2007)

Predator-Prey



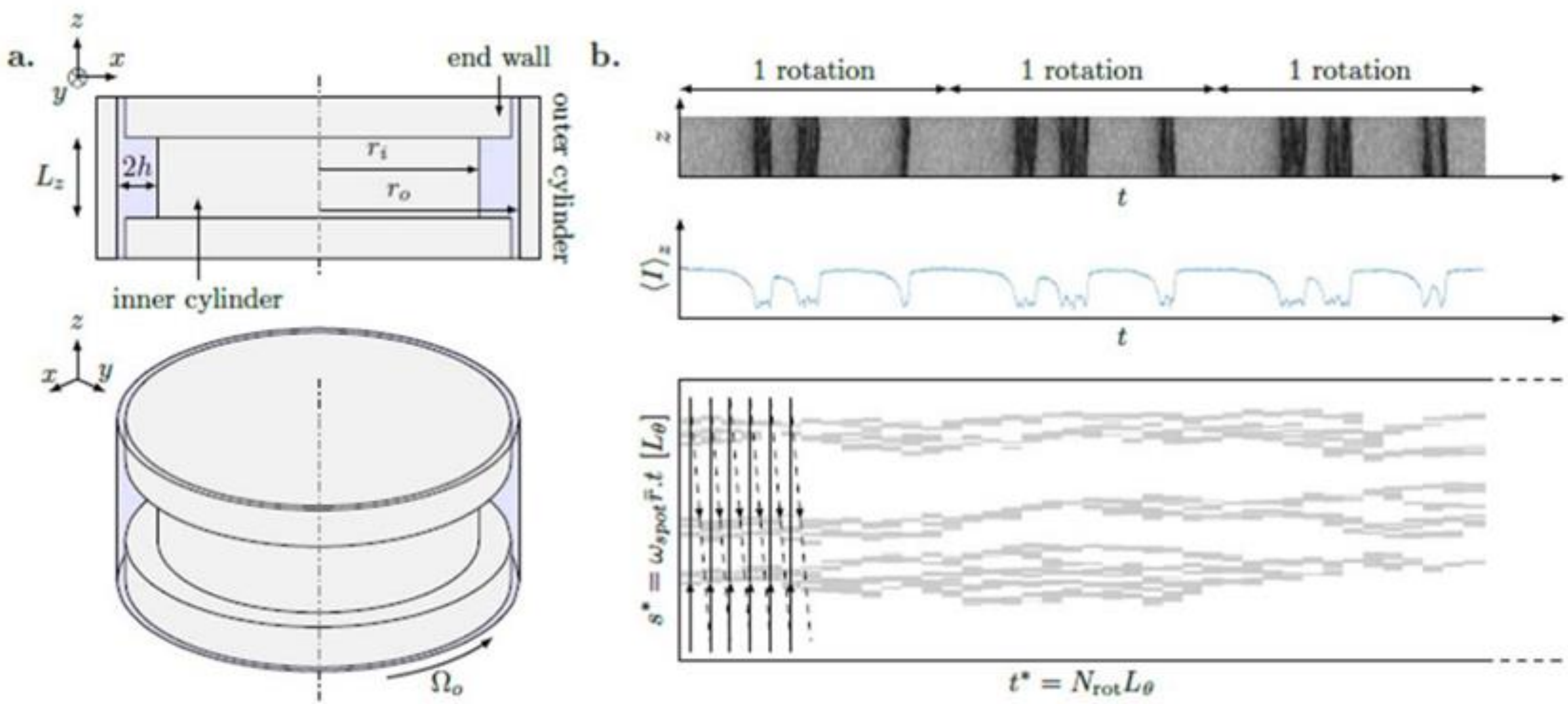
(Pearson Education, Inc., 2009)

Experimental evidence for directed percolation in transitional turbulence in different flow geometries

Turbulence and directed percolation

Fluid between concentric cylinders, outer one rotating

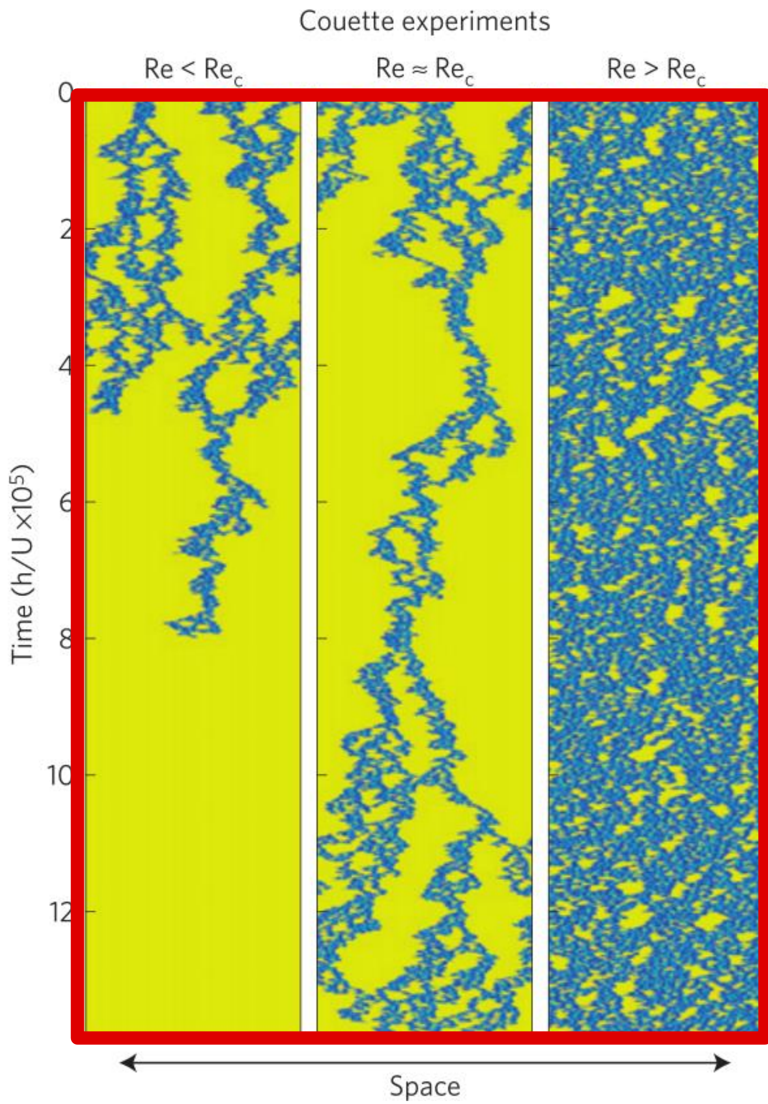
Turbulent patches



Position of turbulent patches changes in time

Lemoult et al., Nature Physics (2016)

Turbulence and directed percolation



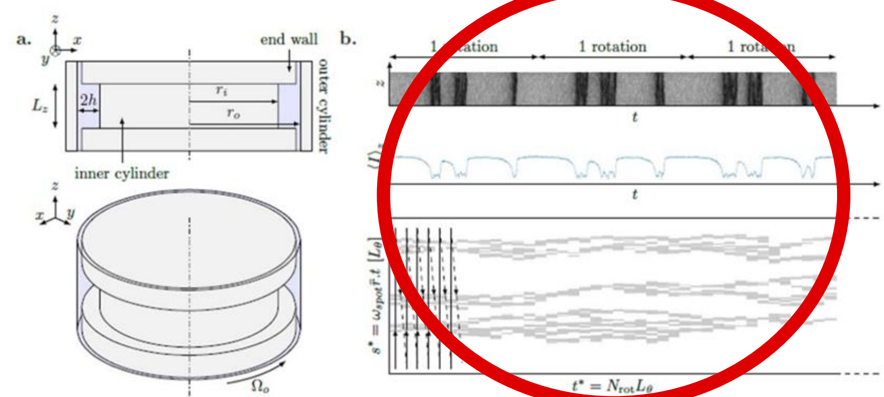
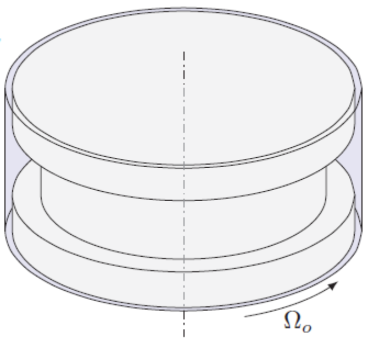


Figure S1: **a** Schematic of experimental set up (not to scale) see text for details. **b** Image of turbulent spots and the conversion to a intensity time series (see text for details).

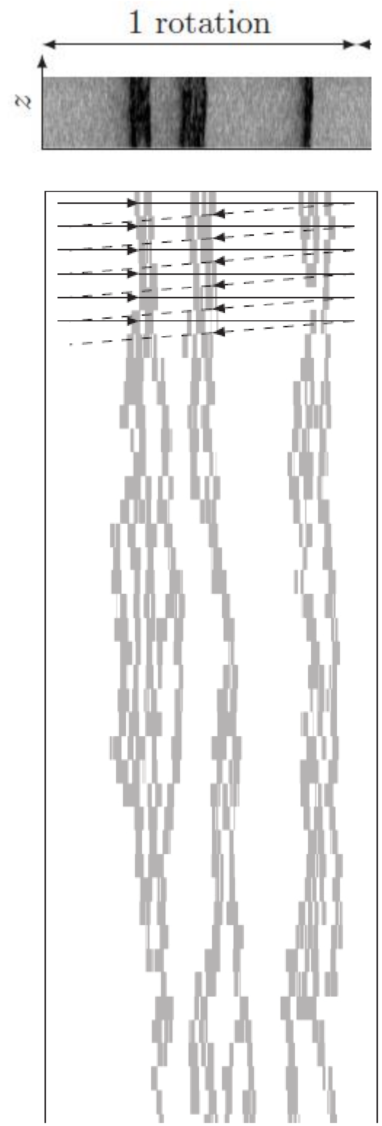
Lemoult et al., Nature Physics (2016)

Directed percolation in turbulence and ecology

Couette

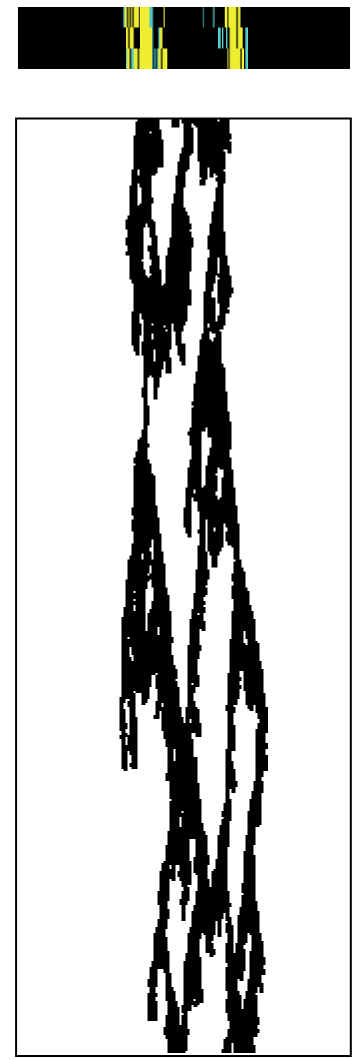


space



time

space



time

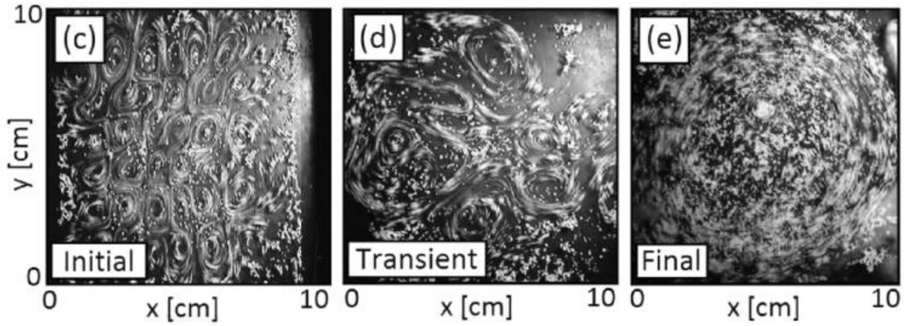
Ecology

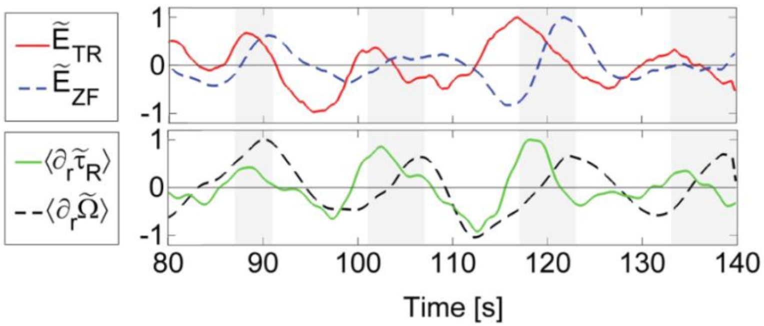
Experimental evidence for predator-prey dynamics in transitional turbulence

Universal predator-prey behavior in transitional turbulence experiments

- 2D magnetized electroconvection

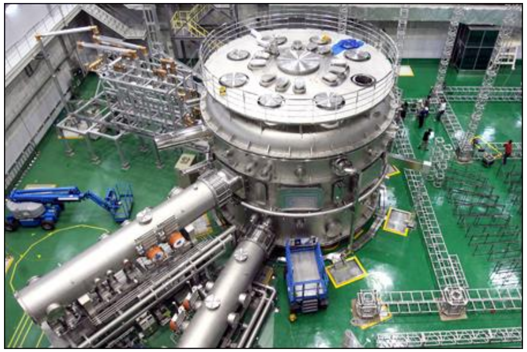
Bardoczi et al. *Phys. Rev E* (2012)

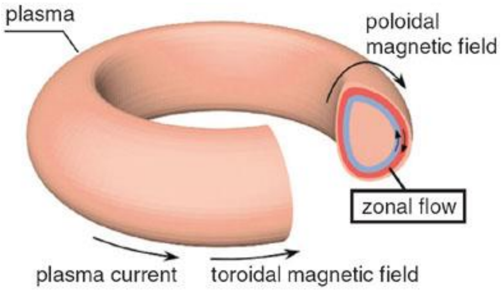


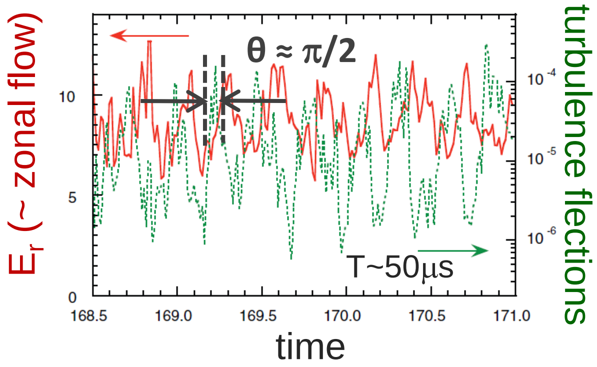


- L-H mode transition in fusion plasmas in tokamak

Estrada et al. *EPL* (2012)







http://alltheworldstokamaks.wordpress.com/gallery-of-external-views/kstar-completed/

Conclusion

- Transition to pipe turbulence is in the universality class of directed percolation, evidenced by:
  - Direct measurement of critical exponents and data collapse universal scaling functions in 1D Couette flow
  - DNS of stress-free Waleffe flow in 2D measures critical exponents and scaling functions
- How to derive universality class from hydrodynamics
  - Small-scale turbulence activates large-scale zonal flow which suppresses small-scale turbulence
  - Effective theory (“Landau theory”) is stochastic predator-prey ecosystem
  - Exact mapping: fluctuating predator-prey = Reggeon field theory = DP near extinction
- Super-exponential behavior of lifetime
  - Turbulence/DP/Predator-prey near extinction shows superexponential lifetime scaling for decay and splitting of puffs

Take-home message

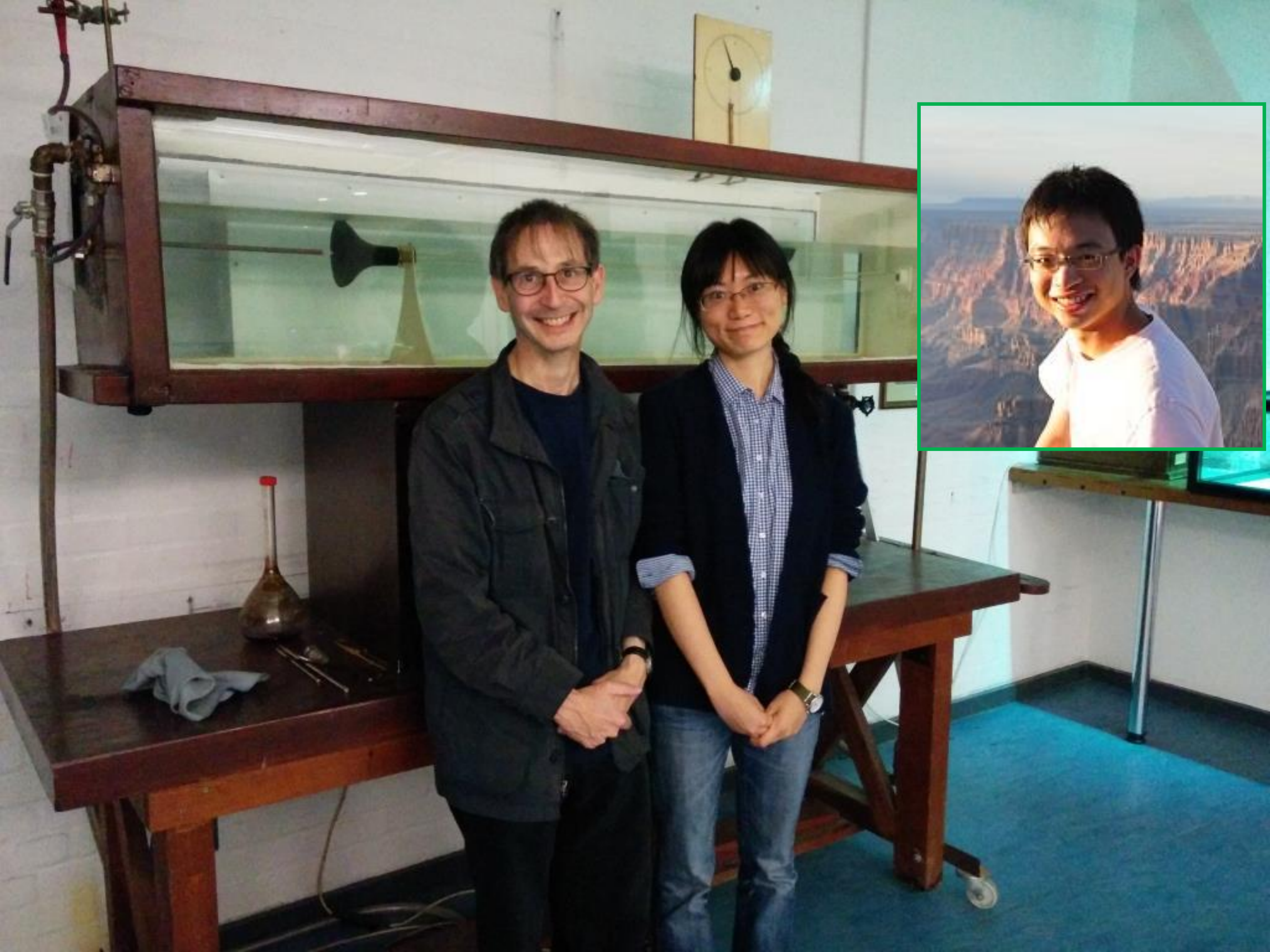
- The Navier-Stokes equations quantitatively obey non-equilibrium statistical mechanics at the onset of turbulence

J Stat Phys (2017) 167:575–594
DOI 10.1007/s10955-016-1682-x

CrossMark

Turbulence as a Problem in Non-equilibrium Statistical Mechanics

Nigel Goldenfeld[1] · Hong-Yan Shih[1]



Turbulence is a life force. It is opportunity. Let's love turbulence and use it for change.

Lucky Numbers 34, 15, 28, 4, 19, 20

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