Block-Structured AMR method for High Accurate Shock Wave Capturing Schemes

o Cheng Liu, Research Institute for Applied Mechanics, Kyushu University, 6-1 Kasuga-koen, Kasuga, Fukuoka 816-8580, Japan, E-mail: liu@riam.kyushu-u.ac.jp
Changhong Hu, E-mail: hu@riam.kyushu-u.ac.jp

Abstract: Parallel Blocked-Structured Adaptive Mesh Refinement (AMR) is an efficient technology to provide high grid resolution with relatively low computer resources. In this short paper, we investigate the efficiency of different AMR decomposition and load balance strategies with a new developed 2D Euler equation AMR solver. The high order weighted essentially non-oscillation (WENO 3rd, 5th, 7th order) scheme with flux-splitting approach is applied for computing problems containing both discontinuities and complex solution features. The performance of the AMR solver is validated by 2-D Riemann problems including double-Mach reflection.

1. Introduction

Numerical simulation of compressible flow is of great important for aerodynamic industry and scientific research. Many successful shock wave capturing scheme have been developed for predicting the sharp discontinuity flow. To eliminate the overshoots and oscillations in the vicinity of the discontinuity, the weighted essentially non-oscillatory (WENO) scheme was proposed in which an adaptive stencil that adapt to the smoothness of the solution is utilized. In this paper, the 3rd, 5th and 7th order WENO scheme are developed and compared.

There are a number of different AMR approaches listed in the literatures [(1)]. Many of them are developed for the usage of the unstructured meshes. Unfortunately, for the unstructured mesh, large degree of indirect memory referencing is required which will cause a low performance on cache-based processors. For the AMR approaches of structured mesh [(1)], we can refine individual grid cells, which can be managed by the tree data structure. It avoid the guard cell overhead problems associated with blocked structured method. But the resulting code is much more complex and difficult to parallelize. Besides, the irregular memory referencing will produce relatively low performance. While the Blocked structured AMR approach [(2)] generate an identical coding environment of each block. Not so many modifications are required to transform from a serial code to parallel AMR code. The parallel data transfer is operated among the blocks, not on the grid, therefore, many advancing parallel algorithm can be performed.

In this paper, we use a high accurate WENO scheme with a flux-splitting approach for the compressible Euler equation. The blocked AMR is applied for mesh refinement. The double-Mach reflection is simulated for validation.

2. Numerical Method

The two-dimensional Euler equations can be written in a conservative form as

$$\frac{\partial \boldsymbol{\Phi}}{\partial t}+\frac{\partial \mathbf{F}}{\partial x}+\frac{\partial \mathbf{G}}{\partial y}=0$$

in which

$$\boldsymbol{\Phi}=\begin{bmatrix}\rho\\ \rho u\\ \rho v\\ \rho E\end{bmatrix}\quad \mathbf{F}=\begin{bmatrix}\rho u\\ p+\rho u^2\\ \rho uv\\ (\rho E+p)u\end{bmatrix}\quad \mathbf{G}=\begin{bmatrix}\rho v\\ \rho uv\\ p+\rho v^2\\ (\rho E+p)v\end{bmatrix}$$

where

$$p=\rho(\gamma-1)\left[E-\frac{1}{2}(u^2+v^2)\right]$$

here ρ, u, v and p are respectively the density, horizontal and vertical velocity and pressure; E denote the internal energy. γ is the ratio of specific heats.

The two-dimensional discretized Euler equation is given as

$$\frac{d\Phi_{i,j}}{dt}+\frac{F_{i+1/2,j}-F_{i-1/2,j}}{\Delta x}+\frac{G_{i,j+1/2}-G_{i,j-1/2}}{\Delta y}=0$$

For the Euler equation, convective flux Jacobian matrixes are

$$\mathbf{A}=\frac{\partial \mathbf{F}}{\partial \boldsymbol{\Phi}}$$

$$\mathbf{B}=\frac{\partial \mathbf{G}}{\partial \boldsymbol{\Phi}}$$

Because the system is hyperbolic, the similarity transformation can be written as

$$\mathbf{A}=\mathbf{S^{-1}}\boldsymbol{\Lambda_A}\mathbf{S}$$

$$\mathbf{B}=\mathbf{T^{-1}}\boldsymbol{\Lambda_B}\mathbf{T}$$

where $\boldsymbol{\Lambda_A}, \boldsymbol{\Lambda_B}$ are respectively the diagonal matrix of the real eigenvalues of $\mathbf{A}$ and $\mathbf{B}$, the columns of $\mathbf{S^{-1}}$ and $\mathbf{T^{-1}}$ are the right eigenvalues of $\mathbf{A}$ and $\mathbf{B}$, and the row of $\mathbf{S}$ and $\mathbf{T}$ are the left eigenvalues of $\mathbf{A}$ and $\mathbf{B}$

$$\boldsymbol{\Lambda_A}=diag(u \quad u \quad u+C \quad u-C)$$

$$\boldsymbol{\Lambda_B}=diag(v \quad v \quad v+C \quad v-C)$$

where C is the sound speed. The expression of $\mathbf{S^{-1}}, \mathbf{T^{-1}}, \mathbf{S}, \mathbf{T}$ can be found in literature [(3)].

The $\boldsymbol{\Lambda_A}, \boldsymbol{\Lambda_B}$ can be split into following

$$\boldsymbol{\Lambda_A}=\boldsymbol{\Lambda_A^+}+\boldsymbol{\Lambda_A^-},\ \ \boldsymbol{\Lambda_B}=\boldsymbol{\Lambda_B^+}+\boldsymbol{\Lambda_B^-}$$

For x- direction,

$$\mathbf{A}=\mathbf{S^{-1}}(\boldsymbol{\Lambda_A^+}+\boldsymbol{\Lambda_A^-})\mathbf{S}$$

The Euler equation become

$$\mathbf{S}\frac{\partial \boldsymbol{\Phi}}{\partial t}+\mathbf{S}\frac{\partial \mathbf{\acute{F}^+}}{\partial x}+\mathbf{S}\frac{\partial \mathbf{\acute{F}^-}}{\partial x}=\frac{\partial \boldsymbol{\Omega}}{\partial t}+\frac{\partial \mathbf{F^+}}{\partial x}+\frac{\partial \mathbf{F^-}}{\partial x}=0$$

where

$$\boldsymbol{\Omega}=\mathbf{S}\boldsymbol{\Phi}, \mathbf{F}^{\pm}=\mathbf{S}\mathbf{\acute{F}}^{\pm}$$

We use the Lax-Friedrichs approach for the flux vector splitting,

$$\mathbf{\acute{F}}^{\pm}=\frac{1}{2}\left[\acute{F}(\boldsymbol{\Phi})\pm\alpha\boldsymbol{\Phi}\right]$$

where

$$\alpha=max\left(\acute{F}'(\boldsymbol{\Phi})\right)=r(\mathbf{A}_i)=max(|u_i|,|u_i+C_i|,|u_i-C_i|)$$

if we use the 5th order WENO reconstruction, the left and right flux can be written as

$$\acute{F}_{i+1/2}^{\pm}=\sum_{k=1}^{3}\omega_k\left(\mathbf{\acute{F}}^{\pm}\right)\mathcal{Q}_k^{\pm}$$

in which ω_k and $\mathcal{Q}_k^{\pm}$ are the weight and interpolation solution from WENO. Note that we should project it into its original formulation.

$$F^{\pm}_{i+1/2} = \mathbf{S^{-1}}\acute{F}^{\pm}_{i+1/2}$$

The 3rd order TVD Runge-Kutta methods for time integration is applied for the equation like $\frac{d\Phi_{i,j}}{dt} = \mathcal{R}(\Phi)$

$$\Phi^{(1)} = \Phi^{n} - \Delta t\mathcal{R}(\Phi^{n})$$

$$\Phi^{(2)} = \frac{3}{4}\Phi^{n} + \frac{1}{4}\Phi^{(1)} - \frac{1}{4}\Delta t\mathcal{R}\left(\Phi^{(1)}\right)$$

$$\Phi^{n+1} = \frac{1}{3}\Phi^{n} + \frac{2}{3}\Phi^{(2)} - \frac{2}{3}\Delta t\mathcal{R}\left(\Phi^{(2)}\right)$$

3. Adaptive Mesh Refinement Strategy

Many applications require consistent data used at the boundaries of different refinement levels. For instance, the present two-dimensional conservative Euler equation, the fluxes entering or leaving a grid cell through a common cell face which is shared with 2 cells of a more refined neighbor. The sum of the fluxes across the 2 smaller faces need to be identical to the flux of its neighbor. In this case, the conservation law treatment is required. We adopt the approach of literature [(4)] by firstly modify the boundary fluxes inside the block boundaries. Then we update the solution around the blocks boundary immediately after the flux correction.

For simplicity, in present AMR simulation, we use the gradient of density to generate the refinement criteria.

4. Numerical Results

For the first validation, a double Mach reflection is simulated with the new developed AMR solver. The problem was initially proposed and studied by Woodward and Colella [(5)]. It has been used extensively in the literature as a test for high resolution schemes. The computational domain is $[0,4]$ and $[0,1]$. The reflecting wall lies at the bottom of the computational domain for $1/6 \le x \le 4$. Initially, a right moving Mach 10 shock is positioned at $x = 1/6, y = 0$ and make a 60 degree angle with the *x*-axis. The ratio of specific heats $\gamma = 1.4$. The results is displayed as following.

We use 3 to 7 levels AMR grid for the simulation. As we can see in Fig. 1, the grids are refined around the shock wave front properly. From the local density distribution in Fig. 2, WENO 5th (Fig. 2(b)) and WENO 7th (Fig. 2(c)) gives a much better resolution for these complicated flow structure than WENO 3rd (Fig. 2(a)) with the same AMR grid resolution.

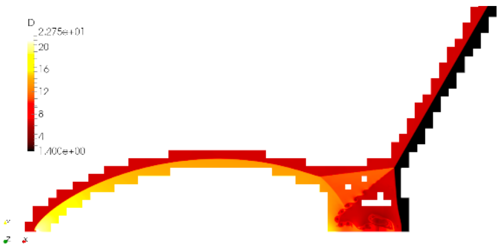
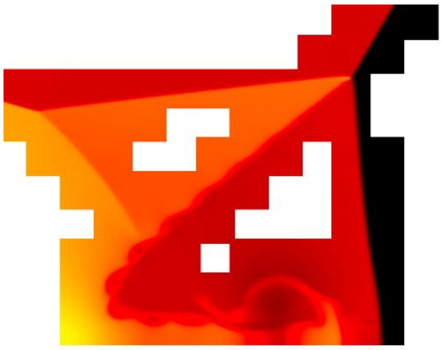


Fig.1 Double Mach reflection problem

5. Conclusion

In present research, the compressible Euler equation is solved with a high order WENO scheme and flux splitting approach to capture the shock wave. Numerical results show that both the blocked structured AMR and 3rd to 7th order WENO scheme works well. Future work will be focused on the study of AMR refinement criteria of the solver.



(a) 3rd WENO reconstruction

(b) 5th WENO reconstruction

(c) 7th WENO reconstruction

Fig.2 Blown-up region of the double Mach reflection problem

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