# Stability Analysis and Transition Prediction of Axi-symmetric Supersonic Boundary Layers over Bodies of Revolution by using Parabolized Stability Equations 

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## 1. Introduction

In this study, stability of compressible axi-symmetric boundary layers over bodies of revolutions is analyzed by using the linear PSE in a generalized curvilinear coordinate system. Transition locations are predicted by using the $e^{\mathrm{N}}$-method with the PSE results. By comparing the predicted transition locations with experimental data, capability and practicality of the prediction procedure used in this study are examined.

## 2. Analysis

The entire procedure is schematically shown in Figure 1.


Fig . 1 Procedure of the prediction of a transition location
The PSE is a set of partial differential equations which is obtained by parabolizing full disturbance equations in streamwise direction as following:

$$
\begin{equation*}
\widehat{D} \psi+\widehat{A} \frac{\partial \psi}{\partial \xi}+\widehat{B} \frac{\partial \psi}{\partial \eta}=\hat{V}_{\eta \eta} \frac{\partial^{2} \psi}{\partial \eta^{2}} . \tag{1}
\end{equation*}
$$

The N -factor is defined as the integration of spatial growth rate along the streamwise coordinate. The N -factors can be calculated for various fixed physical frequencies by using PSE results. The onset of transition location is predicted as the first point where the calculated N -factor exceeds a specified value of N . The predicted transition location is obviously dependent on the choice of N . Therefore, the selection of N based on acceptable data is essential. In this study, $N=6$ for high speed ( $M>4$ ) and $N=8$ for moderate speed ( $1<\mathrm{M}<3$ ) supersonic axi-symmetric boundary layers over bodies of revolution are used for prediction of transition location in wind tunnel experiment condition.

## 3. Results

The stability analysis and transition prediction of the boundary layers over supersonic sharp cone and over Haack Adams bodies of revolution and tangent ogive bodies of revolution are performed.

The supersonic sharp cone boundary layer with free stream Mach number 6 in accordance with the experiment by Horvath (2002) is analyzed. Figure 2 illustrates the measured and predicted transition Reynolds numbers with respect to the free
stream unit Reynolds number. The predicted transition Reynolds numbers with $\mathrm{N}=6$ shows that they are in quite good agreement with the measurement data.


Fig . 2 Comparison of predicted transition Reynolds numbers with measurement data ( $\mathrm{M}_{\mathrm{e}}=5.585$ sharp cone boundary layers )
The boundary layers studied in wind tunnel experiment by Cassels et. al. ${ }^{(14)}$ are analyzed in this work. The predicted transition locations are depicted in figure 3 with the measurement data. It is seen that the present results are in good agreement with the experimental data.


Fig . 3 Comparison of predicted transition Reynolds numbers layer with measured data for boundary layers over Haack-Adams body of revolution with fineness ration 13 ( $M_{\infty}=4.63$ )

## 4. Conclusion

In this study, the stability analysis and transition prediction of supersonic axi-symmetric boundary layers over bodies of revolution are performed by using PSE and the $\mathrm{e}^{\mathrm{N}}$-method. By comparing with experimental data, we have shown that the predicted transition Reynolds numbers are in quite good agreement with the experimental data. We therefore suggest that the use of PSE analysis and $e^{\mathrm{N}}$-method can be a valuable prediction tool to locate transition point over axi-symmetric bodies in supersonic flow.

