Recent advancement of the fully conservative finite difference scheme

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1. Introduction

The fully conservative finite difference scheme for convective terms is recognized as a useful tool for unsteady turbulence simulations like DNS (direct numerical simulation) and LES (large eddy simulation), since it is free of numerical dissipation and offers stable long-term integration. The scheme for incompressible flow is one of commutable convection schemes for divergence, advective, and skew-symmetric forms, which are equivalent provided that the corresponding discrete continuity is satisfied[1]. Note that the schemes for divergence and skew-symmetric forms have primary and secondary conservation properties, respectively, without the aid of the continuity.

Extension of the convection scheme to compressible flow has been attempted by some researchers. However, no convection scheme which is fully conservative for compressible flow was proposed. This was due to the lack of knowledge of commutable convection forms for compressible flow.

Recently, the skew-symmetric form of convective terms and fully conservative convection schemes have been proposed by Morinishi (2009)[2] for compressible flow. In this presentation, the advancement of the convection scheme for compressible flow is reviewed.

2. Forms of convective terms for incompressible flow

First of all, the fundamental concept of the fully conservative convection scheme is illustrated on incompressible flow equations. The velocity field for incompressible flow is constrained by the continuity.

$$(Cont.^I) \equiv \frac{\partial u_j}{\partial x_j} = 0$$

The convective term in the transport equation of variable ϕ can be written as one of the convective forms, that is, the divergence, advective, and skew-symmetric forms:

$$(Div.^I)_\phi \equiv \frac{\partial u_j \phi}{\partial x_j},$$

$$(Adv.^I)_\phi \equiv u_j \frac{\partial \phi}{\partial x_j},$$

$$(Skew.^I)_\phi \equiv \frac{1}{2}\left(\frac{\partial u_j \phi}{\partial x_j} + u_j \frac{\partial \phi}{\partial x_j}\right)$$

These forms are commutable with the aid of the continuity.

$$(Div.^I)_\phi = (Adv.^I)_\phi + \phi\ (Cont.^I),$$

$$(Skew.^I)_\phi = \frac{1}{2}(Div.^I)_\phi + \frac{1}{2}(Adv.^I)_\phi$$

In addition, the divergence form is primary conservative, that is, the form is conservative without the aid of the continuity. The secondary or quadratic conservation property of the skew-symmetric form is demonstrated as follows:

$$\phi\ \left[\frac{\partial \phi}{\partial t} + (Skew.^I)_\phi\right] = \frac{\partial \phi^2/2}{\partial t} + \frac{\partial u_j \phi^2/2}{\partial x_j}$$

The fully conservative convection scheme is the finite difference scheme which satisfies the analytical properties of convective terms in discrete sense[1].

3. Forms of convective terms for compressible flow

Recently, Morinishi (2009)[2] has found that the commutable convective forms for compressible flow can be defined by including the temporal derivative term. This is due to the fact that the continuity for compressible flow contains the temporal derivative term.

$$(Cont.) \equiv \frac{\partial \rho}{\partial t} + \frac{\partial \rho u_j}{\partial x_j} = 0$$

Corresponding divergence, advective, and skew-symmetric forms are defined as follows.

$$(Div.)_\phi \equiv \frac{\partial \rho\phi}{\partial t} + \frac{\partial \rho u_j \phi}{\partial x_j},$$

$$(Adv.)_\phi \equiv \rho\frac{\partial \phi}{\partial t} + \rho u_j \frac{\partial \phi}{\partial x_j},$$

$$(Skew.)_\phi \equiv \sqrt{\rho}\,\frac{\partial \sqrt{\rho}\,\phi}{\partial t} + \frac{1}{2}\left(\frac{\partial \rho u_j \phi}{\partial x_j} + \rho u_j \frac{\partial \phi}{\partial x_j}\right)$$

The divergence, advective, and skew-symmetric forms for compressible flow are equivalent if $(Cont.) = 0$ is satisfied.

$$(Div.)_\phi = (Adv.)_\phi + \phi\ (Cont.),$$

$$(Skew.)_\phi = \frac{1}{2}(Div.)_\phi + \frac{1}{2}(Adv.)_\phi$$

The divergence form is primary conservative without the aid of the continuity. The secondary conservation property of the skew-symmetric form is demonstrated as follows:

$$\phi\ (Skew.)_\phi = \frac{\partial \rho\phi^2/2}{\partial t} + \frac{\partial \rho u_j \phi^2/2}{\partial x_j}$$

Based on the forms of convection and the analytical properties, the fully conservative convection schemes for compressible flow have been proposed by Morinishi (2009)[2].

Bibliography

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(2) Y. Morinishi, Skew-symmetric form of convective terms and fully conservative finite difference schemes for variable density low-Mach number flows, J. Comput. Phys., (2009), doi:10.1016/j.jcp.2009.09.021.