

THE ACOUSTIC INSTABILITY OF A VORTEX FLOW AFFECTED BY SOUND WAVES

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The propagation of sound waves in a finite vortex is investigated by numerically solving the linearized Euler equations in two dimensions. Two types of vortices are considered: homentropic, i.e., with uniformly distributed entropy in space, and non-homentropic, i.e., with a non-constant distribution of entropy in the vortex core. In the latter case, the results reveal an instability of the acoustic field, which is caused by "entropy acoustic waves" generated in the vortex region by the incident sound waves.

Statement of the problem. In the present paper the scattering of sound waves by a finite vortex is numerically investigated by solving the Linearized Euler Equations (LEE). The base flow is represented by an axisymmetric isolated vortex, where the distribution of state parameters in the radial direction is taken so as to follow the Euler equations.

Two vortical flows are considered here. One is a homentropic vortex, where the pressure and density are related such that the entropy has a constant distribution in space. The other is a non-homentropic vortex with a non-constant distribution of the entropy. For those cases, the velocity field of the base flow is taken, which decays exponentially fast far away from the vortex centre. Such an distribution of the angular velocity has been used in several recent papers, for example [1].

The calculation domain is a square with a non-dimensional length $L=10$, where the radius of the vortex core is taken as the reference length. Sound waves are irradiated by a speaker or vibrator (5% of L) that is located at the middle of the left boundary. All boundaries of the computational domain except for the speaker are treated as non-reflecting. A grid is used in this calculation, which consists of 201 evenly spaced cells in each direction. The numerical method is an extension of the Godunov method to the LEE, which employs the exact solution to the Riemann problem [2] to approximate acoustic numerical flux.

Main results. Calculations are carried out under different conditions that are characterised by two parameters: the intensity of the base vortex μ , which is proportional to the maximal velocity in the vortex, and parameter δ that equals the ratio between the speaker wavelength λ and the vortex radius, $\delta=\lambda/a$, a being the radius of the vortex. Analysis of these calculations allows us to infer the following.

-) The scattering of sound waves by the homentropic vortex does not induce any instabilities, and the computed scattered acoustic field exhibits same characteristics as predicted by both analytical and another numerical methods [1].
-) The acoustic field in the case of non-homentropic vortex becomes unstable; that is, an unbounded increase in the amplitude of all parameters (such as acoustic pressure, velocity, and density) is observed; this instability is caused by "entropy acoustic waves" generated in the vortex core by incident waves.
-) This instability is accompanied by the formation of a regular structure in the spatial distribution of acoustic parameters in the region of the vortex, where the distribution is distinctly periodical in the angular variable;
-) Depending on the speaker's frequency and the vortex intensity, there are found several modes of instabilities characterised by different number of waves in the circumferential direction.
-) There is a time delay in the instability excitation; the unstable mode appears in the acoustic field after a lapse of some time

period after the speaker begins to emit sound waves. Up to this time any unstable processes in the acoustic field are not observed.

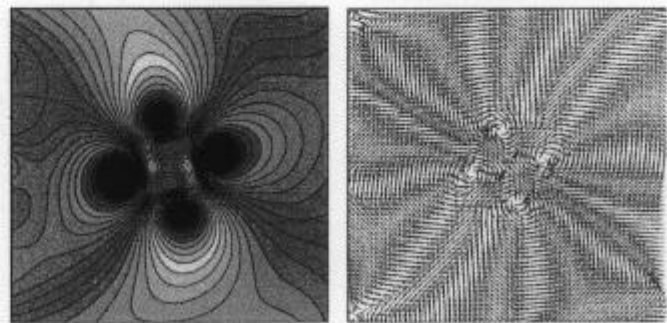


Fig. 1. Acoustic pressure field (left) and velocity vector (right) for the scattering of non-homentropic vortex by sound waves.

A typical example of such an instability is shown in Fig. 1. Here the acoustic pressure (left figure) and the acoustic velocity vector (right figure) are depicted for the scattering of a non-homentropic vortex with $\mu=0.8$ that corresponds a value of the maximal Mach number of 0.25, where the entropy s is taken as $s=1+\theta \exp(-r^2)$ with $\theta=-0.5$. Sound waves irradiated by the vibrator have $\delta=4.72$ and an amplitude of 0.001. The pressure contours are drawn for the range of values between -0.003 and 0.003 with a uniform displacement.

As seen from these figures, a regular 4-leafed rapidly growing structure is formed in the vortex core. This is characterised by intensive amplification of the amplitude of all acoustic parameters; the maximal amplitude of the acoustic pressure in the vortex exceeds that of the incident waves more than 3 times by the moment shown in Fig. 1. The pressure field has a distinct pattern quite different from that of incident waves, where the pressure varies periodically from positive values to negative and vice versa.

The acoustic velocity vector clearly reveals that this instability is accompanied by the formation of a regular structure of singular points, which consists of four focus-type points located along the vortex periphery and one saddle-type point in the vortex centre for the considered flow conditions. Once it appears, this structure becomes self-exciting, and does not vanish, even if the vibrator is switched off.

References

- [1] Colonius, T., Lele, K. and Moin, P., "The scattering of sound waves by a vortex: numerical simulations and analytical solutions", *J.Fluid.Mech.*, **260**(1994), pp. 271-298.
- [2] Men'shov, I. and Nakamura, Y., "On implicit Godunov's method with exactly linearized numerical flux", *Computers & Fluids*, **29**(2000), pp.595-616.