Large Eddy Simulation Of Flow Past A Two-dimensional Hill

Sankara N.Vengadesan[1)] and Akihiko Nakayama[2)]
[1)]Research Associate, Email: vengades@kobe-u.ac.jp, [2)]Professor, Email: nakayama@kobe-u.ac.jp
Graduate School of Science and Technology, Kobe University, Kobe 657-8501, Japan

Turbulent flow past an idealized two-dimensional hill with two different slopes is simulated by LES technique. Simulations are performed by both conventional and dynamic Smagorinsky models. Standard Smagorinsky case is simulated with three different types of near-wall treatments. Results of simulations by log-law modified to take into account for local and instantaneous pressure gradient effects are closer to the experimental data in terms of mean velocity and wake size.

1. INTRODUCTION

Wind flow around hill and over complex terrain is of great interest in engineering applications like transport and dispersion of pollutants in the atmosphere, agro-meteorological study, construction of wind mills and airport etc., Practically, an infinite number of situations are possible due to varieties of hill geometry, arrangements, and approaching flow conditions. Various experimental measurements have been reported for different configuration of the hills at different Reynolds numbers [1-5]. Numerical predictions have been performed [4-8], invariably all the calculations so far are of RANS type and no Large Eddy Simulation (LES) study has yet been reported yet.

In LES method, large-scale motion is resolved by discrete computational grid and directly computed by numerical method and small-scales of motions are modeled. While this technique has been proved to be very successful in simulating simple flows over smooth boundaries and is considered to be a promising tool in engineering, its application to flows in natural environment is not quite straightforward, as boundaries are generally rough and their geometry is very complex.

In the present work, we perform Large Eddy Simulation of flow past an isolated two-dimensional hill with particular emphasis on investigation appropriate near wall boundary condition to be applied to simulation of high *Re* flows over natural terrain. The hill is of bell shape defined by an analytical expression. Experimental measurements are available at moderately high Reynolds number. Two different LES models – standard and dynamic Smagorinsky and three types of near wall boundary conditions are considered for simulation. Results are analyzed in terms of mean velocity and turbulence quantities, influence of boundary conditions are elucidated.

2. DESCRIPTION OF THE HILL GEOMETRY AND FLOW CONDITION

The flow configuration considered is that past an isolated hill (Fig.1), a smooth two-dimensional topography, defined by an analytical expression $\frac{z_G}{H}=\frac{1}{1+(x/nH)^4}$, where z_G is the elevation of the ground at horizontal position x, and H is the height of the hill. We consider this test case with two maximum values of the slope angle viz. 15 degree and 25 degree, measured from the horizontal direction, determined by values of n. Table 1 gives details of index n, hill height H and the respective slope angle. This flow has been subjected to a detailed experimental study and experimental results are available in Nakayama and Yokota[9]. Mean velocity and turbulent stresses have been measured for the Reynolds number based on the oncoming reference velocity U_{ref} and H of 13000. This is relatively a gentle topography and no flow separation is reported.

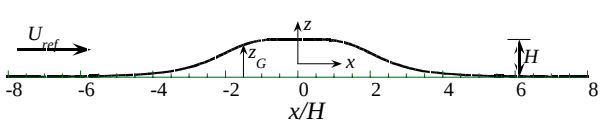


Fig.1 Flow configuration

Table 1. Hill Geometry parameters

H (cm)	n	Maximum slope angle (degrees)
10	23	25
5	20	15

3. NUMERICAL METHODS

The basic equations used in the present LES are three-dimensional, time dependent, Navier-Stokes equations, filtered in order to separate the large scale and the small-scale motions. We consider isothermal incompressible flow and solve the filtered governing equations along with closure subgrid-stress model. As the focus of the present work is a study on the influence of boundary conditions, governing equations are not described here and they can be found in Nakayama and Noda[10] and standard text books [11,12,13]. LES model chosen is the conventional Smagorinsky model in which the turbulent stress, R_{ij} is modeled as

$$R_{ij}=\frac{2}{3}k_s\delta_{ij}-2\nu_G S_{ij} \tag{1}$$

where, k_s is the subgrid turbulent kinetic energy, δ_{ij} is the Kronecker delta, ν_G is the subgrid eddy viscosity and S_{ij} is the strain tensor. The eddy viscosity ν_G is modeled by

$$\nu_G=(C_S\Delta)^2\left[\frac{\partial u_i}{\partial x_j}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)\right]^{1/2} \tag{2}$$

where, Δ is the grid size defined by the geometric average of the grid spacings in three directions, $(\Delta x_1\Delta x_2\Delta x_3)^{1/3}$, u_i is the spatially filtered velocity component in the streamwise -x_i direction, u_2 in the spanwise – x_2 direction and $u3$ in the cross-stream –x_3 direction, i.e $(x_1,x_2,x_3)=(x,y,z)$; $(u_1,u_2,u_3)=(u,v,w)$. C_s is the model constant for which we use the value of 0.13 in the case of Standard Smagorinsky model. We also use the dynamic procedure to determine the value of C_s due to Germano et al.[14] with Lilly's[15] modification.

(1) Calculation domain and grid

The computational region covers the test flow shown in Fig.1, from about 8.5H in the upstream and 14H in downstream in the streamwise direction, 7H in the cross stream-wise and 4H in the spanwise direction. A rectangular grid is used, which is uniformly spaced in the spanwise direction. In the streamwise direction, points are closely spaced (90 points) within 4H on either side from the hill summit, stretched with a factor of 1.038. In the cross stream-wise direction, the first point from the ground is placed at 0.03H near the bottom of the wall, stretched with a factor of 1.05 upto 0.5H and then compressed with a factor of 0.95 upto 1.5H and then placed non-uniformly with stretching factor of 1.1 until the end. This grid

distribution gives $z^+=zu_\tau/\nu$ of the first node about 20 on the top of hill and about 15 at x/H=4 and thus viscous layer are not resolved. The total grid size is 128x61x21.

The curved boundary is represented by cartesian co-ordinate system with staggered mesh arrangement. The boundary conditions are applied at the mesh points closest to the real boundary, but not exactly on it. In order to find the influence of the approximate position of the boundary, example of calculated velocity vectors along with grid and test case geometry between two streamwise stations are shown in Fig.2. This figure shows that there is no such thing as step corners due to the approximation. Small deviation of the vectors from the direction tangent to the local boundary surface is seen, but this does not influence the results on the whole.

(2) Numerical schemes

We solve the governing equations by a finite difference procedure. Non-linear convective terms in the equations are discretised by a third order upwind differencing, (UTOPIA) to avoid stability problems and viscous terms are discretised by second-order accurate central differencing scheme. Inflow conditions for the streamwise velocities are adopted from experimental data. Radiation outflow condition is applied at the downstream boundary. The periodic boundary conditions are used for the spanwise direction. In the cross flow direction, the nonslip boundary conditions are applied on the ground surface and slip conditions are applied on the top boundary. HSMAC iteration scheme is used for calculating pressure. Time advancing of the momentum equations is done by a second-order accurate explicit, Adams-Bashforth method. Performance of the code had been assessed earlier for flow past a bluff body and for the curved geometry by Nakayama and Noda[10]. All the calculations are performed with the non-dimensional time step, dtU_{ref}/H of 0.001. All calculations have been allowed to settle down until 40 non-dimensional time units, and then statistical averages over the next 40 non-dimensional time units are obtained that are presented below.

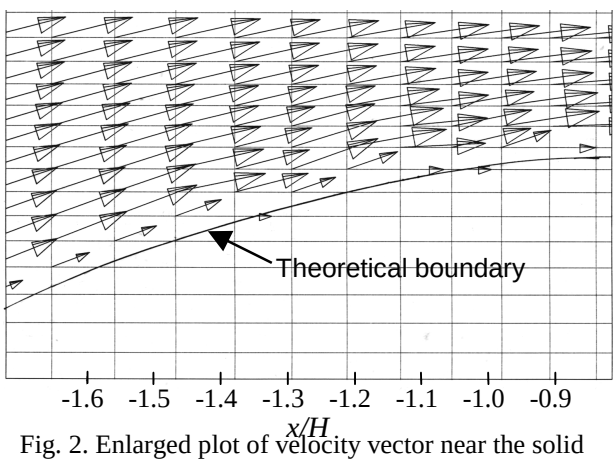


Fig. 2. Enlarged plot of velocity vector near the solid boundary

(3) Wall boundary conditions

In wall-bounded flows, the only correct boundary condition at the surface is the no-slip condition, but this requires calculations up to the wall with sufficient grid resolution. However, as the Reynolds number increases, boundary layer thickness decreases, resulting in requirement of large number of grid points. In RANS type simulation, wall-function approach is used as one method of way out to meet this condition. But in LES the problem is severe as pointed out by Spalart et.al.[16] and no definite solution has been proposed yet. We perform calculation for the present test case, with non-slip boundary condition as a baseline solution to compare and this case is referred to as Case A.

Hino and Okumura[17] have performed flow over a wavy wall by assuming single-layer linear distribution for the velocity. This approximation is good only when viscous sublayer can be resolved. When the laminar sublayer cannot be resolved by the computational grid, artificial boundary condition may be applied at some distance from the wall. In LES, this technique is recently referred to as "Off the wall" boundary condition, (Cabot[18]). As one method, Werner-Wengle[19] proposed instantaneous two-layer linear-power law velocity distribution. This has been used quite extensively in many LES calculations, reported by Rodi et.al.[20]. However, they cannot be used in separated flows and non-equilibrium flows. This two-layer model is modified into three-layer linear-loglaw version in the format given by Von Karman[21], to specify the boundary conditions for the velocities in the tangential directions, at the first point from the wall. Nakayama et.al.[22] have tested the validity of this boundary condition, for LES of flow over an isolated hill at Re=50000. The maximum slope angle of this is 45 degrees from the horizontal direction and according to the experiment, the flow separates on the lee side and then reattaches. They found that the prediction with log-law boundary condition was closer to the experimental data compared with calculations without it. However, they indicate that the use of the log-law was too dissipative to show significant turbulent fluctuations.

We test once again the three-layer linear-log law in the present model and this is referred to as Case B. In this method, an approximation to a wall law given by the following equation is used.

$$u^+ = z^+, \qquad 5 > z^+ > 0 \tag{3}$$
$$u^+ = 5 \ln z^+ - 3.05, \qquad 30 > z^+ \geq 5 \tag{4}$$
$$u^+ = 2.5 \ln z^+ + 5.5, \qquad z^+ \geq 30 \tag{5}$$

where, $z^+=zu_\tau/\nu$ and $u^+=u/u_\tau$ are the non-dimensionalised vertical distance and velocity respectively. The friction velocity, u_τ is calculated from these equations with the velocities at the second point from the wall.

Natural terrain is subjected to wavy topography and the flow past it is subjected local acceleration and deceleration due to pressure gradients and resistance due to roughness of boundary. In Case C, we use the log-law modified to include local and instantaneous pressure gradient effects in the format given by Wilcox[23], to specify velocities at the first point from the wall. The wall function for the velocity modified to include pressure gradient effects is given as:

$$u^+ = z^+, \qquad 10 > z^+ > 0 \tag{6}$$
$$u^+ = 2.5 \ln z^+ + 5.5 - C_{pc}\, z^+ P^+, \qquad z^+ \geq 10 \tag{7}$$

where, P^+ is the dimensionless pressure-gradient parameter defined by $P^+ = \frac{\nu}{u_\tau^3}\frac{dP}{ds}$, where s is the distance along the boundary. The above equation is proposed for mean velocity profile. In order to apply it to the instantaneous velocity, the coefficient C_{pc} for the pressure gradient term is set to 0.005.

Table 2 gives the details of different computational runs and keys used to refer to them.

4. RESULTS AND DISCUSSION

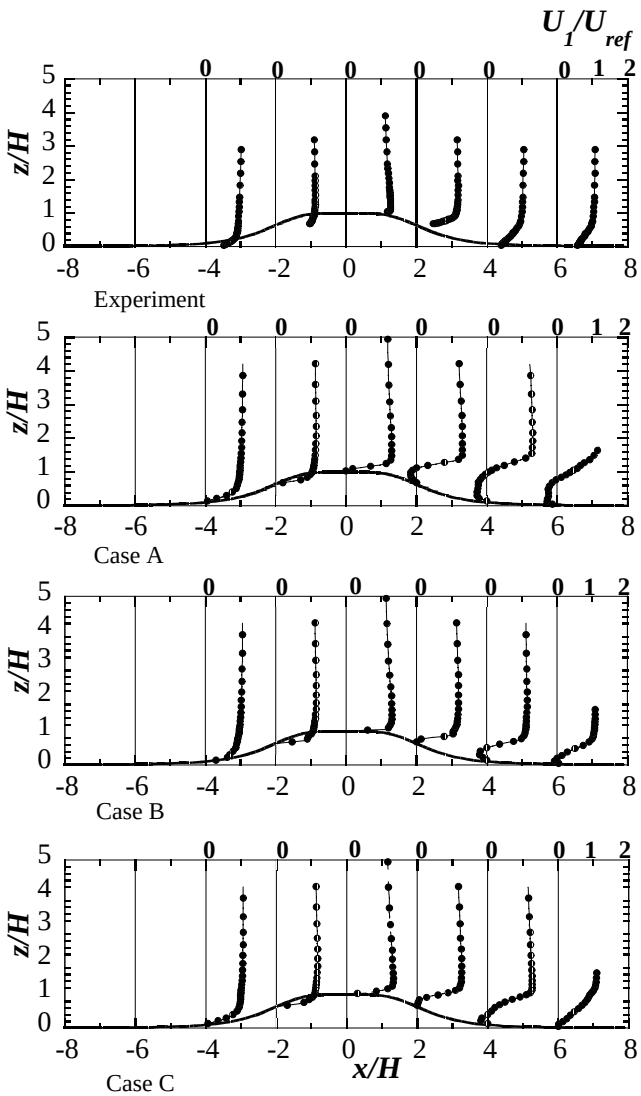
Computational cases Case A to Case C is run for hill with maximum slope angle 25 degrees and all the four cases are run for hill with maximum slope angle for 15 degrees.

Table 2. Computational runs

Smagorinsky Model	Boundary condition	Key
Standard	Non-slip	Case A
Standard	Conventional log-law	Case B
Standard	Log-law with pressure gradient effects	Case C
Dynamic	Non-slip	Case D

(1) Mean velocity

Profiles of time averaged streamwise velocity component, U_1 at specified streamwise stations computed using different boundary conditions, by two Smagorinsky models and experimental results are plotted in Fig.3 (a) for hill with 25 degrees and Fig. 3(b) for hill with 15 degrees. The inflow velocity profile for the calculations at station *x/H*=-4 and *x/H*=-6 for 25 degrees and 15 degrees respectively are taken from that of experiment. At *x/H*=0, experimental results show that the flow accelerates just near the top of the hill. At the same station, calculation with the nonslip boundary condition (Case A) shows the development of the boundary layer and the maximum velocity is drastically under-predicted. Calculations using the log-law boundary condition (Case B) show a thinner boundary layer and results by the modified log-law (Case C) are seen closer to the experiment. As it can be observed, Case A shows separation at *x/H*=2 and *x/H*=4 for 25 degrees and 15 degrees hill respectively and predicts a large re-circulation zone at further downstream. There the results using the modified log-law (Case C), in which the effects of one of the locally changing parameters are included, show trends that are in better agreement with the experiment. This implies that to improve simulation results for a complex topography, one needs to include influence of locally and temporally changing parameters



(a) Hill with slope angle 25 degrees

Fig.3. Velocity profiles along selected cross-stream sections

(2) Turbulence quantities

Calculated shear stresses are shown in Fig. 4 at two streamwise stations – *x/H=0* and *x/H=6* along with the experimental data. On top of the hill, all three cases grossly under-predict the distribution and at farther downstream, there is a improvement in the predictions. At *x/H*=0, prediction with the non-slip boundary condition case is closer to the experimental values. This can be attributed to the fact that early massive separation caused is responsible for turbulent production. Prediction using the conventional log-law is the worst among and this may be interpreted that in this case very small turbulence is produced. At *x/H*=6, the results of Case A shows a large negative shear stress which is also due to the large separation. Here the modified boundary condition appears to give results closer to the experiment.

Reynolds normal stresses are shown in Fig.5 at *x/H=6* station. Case D results are not included as they are found to be very much out of range. Prediction by non-slip boundary condition shows the same behavior as noticed earlier. Conventional log-law boundary condition case results are in between and results of proposed modification case under predict the stresses. Results on the whole indicate that in order to improve prediction, some kind of mechanism to produce and retain turbulence is to be incorporated in the model or in the simulation procedure.

5. CONCLUSIONS

Large Eddy Simulation of flow past an idealized two-dimensional hill with maximum slope angle of 25 degrees and 15 degrees at moderate *Re* number has been performed. Both Standard and dynamic type Smagorinsky model along with three different types of near-wall boundary conditions are considered. At first, two existing boundary conditions – non-slip and conventional log-law assumption are studied and their limiting behavior is elucidated. A modification in the conventional log-law to include local and instantaneous pressure gradient effects that reflects local acceleration and deceleration due to changing topography is proposed. This method improved the simulation results in terms of wake size and mean velocity. But, predictions of turbulence quantities are not satisfactory. It is found from the present study that for LES of the flow near ground, where the topography is never smooth and quite undulating, by considering other parameters such as the rate of temporal change and three-dimensionality, further refinements may be possible.

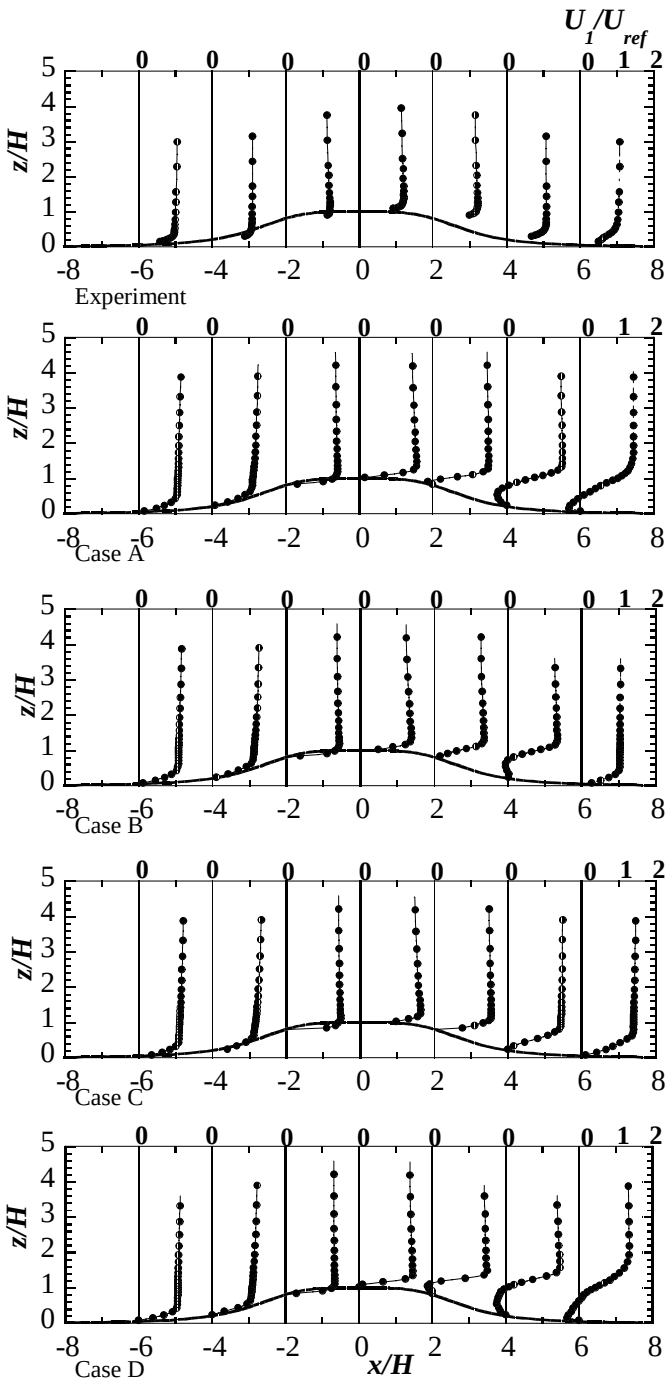


Fig.3. Contd., (b) Hill with slope angle 15 degrees

References

1) Baskaran, V., Smits, A.J., and Joubert, P.N.: A turbulent flow over a curved hill, Part 1. Growth on an internal boundary layer, *J.Fluid Mech.*, Vol. 182, pp.47-83, 1987.

2) Hunt, J.C.R., Leibovich, S., and Richards, K.J.: Turbulent shear flow over low hills, *Royal Metrological Society*, Vol., pp. 1435-1470, 1988.

3) Almeida, G.P., Durao, D.F.G., and Heitor, M.V.: Wake flows behind two-dimensional model hills, *Experimental Thermal and Fluid Science*, Vol.7, pp.87-101, 1993.

4) Ferreira, A.D., Lopes, A.M.G., Viegas, D.X., and Sousa, A.C.M.: Experimental and numerical simulation of flow around two-dimensional hills, *J. Wind Eng and Ind. Aerodyn.*, Vol.54/55, pp.173-181, 1995.

5) Kim, H.G., Lee, C.M., Lim, H.C., and Kyong, N.H.: An experimental and numerical study on the flow over two-dimensional hills, *J.Wind Eng. and Ind. Aerodyn.*, Vol.66, pp.17-33, 1997.

6) Kobayashi, M.H., Pereira, J.C.F., and Siqueira, M.B.B.: Numerical study of the turbulent flow over and in a model forest on a 2D hill, *J.Wind Eng. and Ind. Aerodyn.*, Vol.53, pp.357-374, 1994.

7) Ying, R., and Canuto, V.M.: Numerical simulation of flow over two-dimensional hills using a second-order turbulence closure model, *Boundary Layer Metrology*, Vol. 85, pp.447-474, 1997.

8) Peng, S., and Davidson, L.: New two-equation eddy viscosity transport model for turbulent flow computation, *AIAA J.*, Vol.38, n7, pp.1196-1205, 2000.

9) Nakayama, A., and Yokota, D.: The influence of pressure-gradients and curvature on the rough-wall boundary layers, *Proc.2000 Meeting of JSFM*, pp.839-840, 2000 (in Japanese).

10) Nakayama, A., and Noda, H.: LES simulation of flow around a bluff body fitted with splitter plate, *J. Wind Eng. and Ind. Aerody.*, Vol.85, pp.85-96, 2000.

11) Wilcox, D.C.: *Turbulence modeling for CFD*, 2nd ed., DCW industries, 1998.

12) Voke, P.R., Sandham, N.D., and Kleiser, L.: Direct and Large-Eddy Simulation III, *Proc. of Issac Newton Institute Symposium/ERCOFTAC Workshop*, Kluwer Academic Publishers, 1999.

13) Peyret, R., and Krause, E.: *Advanced Turbulent flow Computations*, CISM courses and lectures, No.395, Springer Wein, NewYork, 2000.

14) Germano, M., Piomelli, U., Moin, P., and Cabot, W.H.: A dymaic subgrid scale eddy viscosity model, *Phys. Fluids A*, Vol.3, pp.1760-1765, 1991.

15) Lilly, D.K.: A proposed modification of the Germano subgrid-scale closure method, *Phys. Fluids A*, Vol.4, pp.633-635, 1992.

16) Spalart, P.R., Jou, W.H., Strelets, M., and Allmaras, S.R.: Comments on the feasibility of LES for wings, and on a Hybrid RANS/LES approach, *Advances in DNS/LES, Proc. 1st AFOSR Inter. Conf. on DNS and LES*, Greyden Press, 1997.

17) Hino, M., and Okumura, T.: Coherent structure of turbulent flow over wavy walls, *Proc. 9th symp. of Turbulent Shear Flows*, pp.14-3-1 – 14-3-4, 1993.

18) Cabot, W.: Wall models in Large Eddy Simulation, NASA Ames/Stanford Center for Turbulence Research, *Annual Research Briefs*, pp.657-681, 1997.

19) Werner, H., and Wengle, H: Large-Eddy simulation of turbulent flow over and around a cube in a plate channel, *Proc. 8th Turbulent Shear Flows*, Munich, pp.19-4-1 – 19-4-6, 1991.

20) Rodi, W., Ferziger, H.H, Breuer, M., and Pourquie, M.: Status of Large Eddy Simulation-Results of a Workshop, *J.Fluids Engg.*, Vol,119, pp.248-262, 1997.

21) Von Karman, T.: The analogy between fluid friction and heat transfer, *Trans*. ASME, Vol.61, pp.705-710, 1939.

22) Nakayama, A., Miyashita, K., and Maeda, K.: Comparison of numerical analysis of flow past topography by time-dependent RANS and LES simulation, *Annual J.Hydraulics Engg.*, JSCE, Vol.44, pp.43-48, 2000 (in Japanese).

23) Wilcox, D.C.: Wall matching, A rational alternative to wall functions, *AIAA paper* 89-611, 1989.

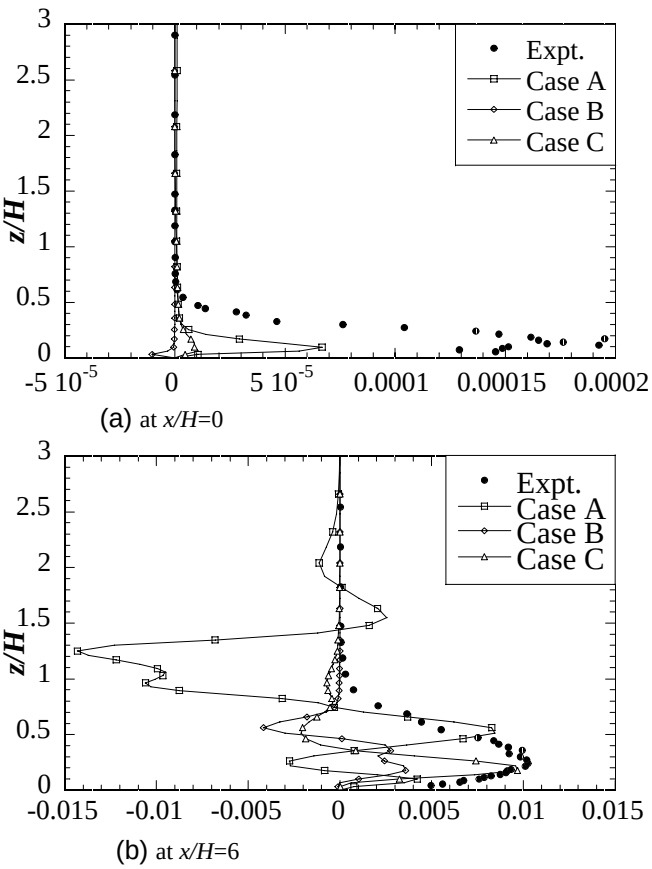


Fig.4. Shear stress distribution for hill with slope angle 25 degrees

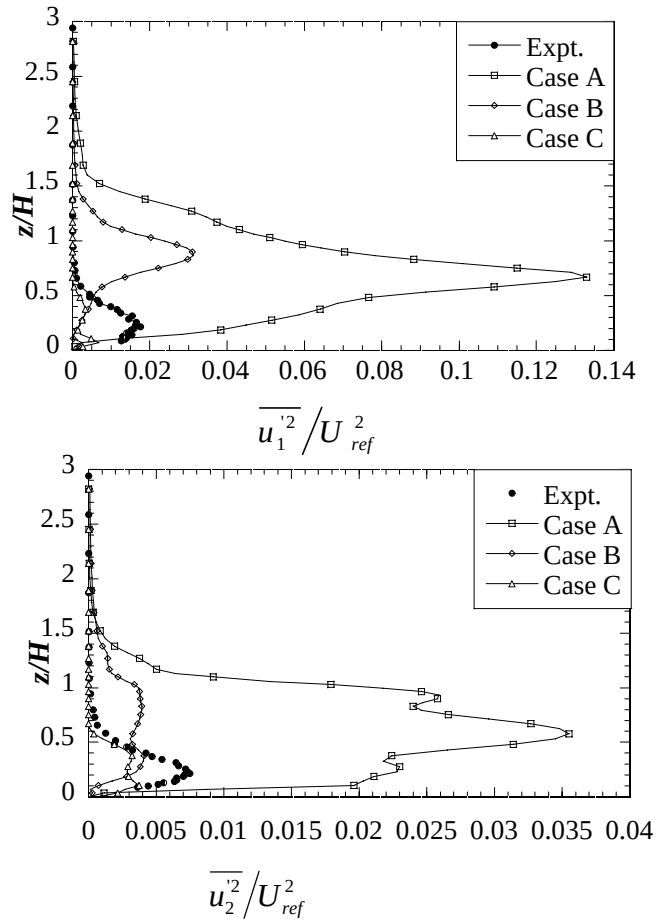


Fig.5. Reynolds stress distribution at *x/H*=6 for hill with slope angle 15 degrees