保存・無振動セミラグランジュスキーム

Conservative and oscillation-less Semi-Lagrangian Schemes

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Abstract: A type of semi-Lagrangian schemes was proposed to compute conservative transportation equation. The mass conservation and oscillation-suppressing properties are enforced by imposing proper constraining conditions to the reconstruction of interpolation function. Excellent numerical results were achieved for both linear and non-linear scalar conservation laws.

1.Introduction

Semi-Lagrangian methods that conserve exactly the transported physical field have been recently developed in [1,2] by using the CIP (Constrained Interpolation Profile) concept.

In this paper, we propose another class of schemes called CIP-CSL3 (Constrained Interpolation Profile - Conservative Semi-Lagrangian scheme with 3rd-order polynomial function). The CIP-CLS3 schemes are constructed from a cubic polynomial. In addition to the conservation constraint used in the CIP-CSL2 , the slope (first-order derivative) of the interpolation function at the middle point of a mesh cell is also introduced as another constraint on the interpolation function. The slope at the cell center can be easily approximated from a reconstruction procedure and allows manipulations, i.e. slope limiters, to make the interpolation oscillation-less.

2. The schemes

The model equation to be considered is a transportation equation in one dimension

$$\frac{\partial f}{\partial t} + \frac{\partial uf}{\partial x} = 0 \tag{1}$$

where t refers to the time, x the spatial coordinate, u the characteristic speed and f the transported quantity.

We make use of a piecewise cubic polynomial function $F_i(x)$.

As in the CIP-CSL2 method, a constraint for the conservation of cell-integrated average is imposed as

$$\frac{1}{\Delta x_{i-1/2}} \int_{x_{i-1}}^{x_i} F_i(x) dx = \rho^n_{i-1/2} \tag{2}$$

where $\Delta x_{i-1/2} = x_i - x_{i-1}$.

Another constrained condition for the interpolation construction is imposed on the first-order derivative at the middle point of the cell as

$$\frac{dF_i(x)}{dx} = d^n_{i-1/2}. \tag{3}$$

In terms of $f^n_{i-1}, f^n_i, d^n_{i-1/2}$ and ρ^n_i, the cubic polynomial $F_i(x)$ can be completely determined. The numerical solution of f at time step $n+1$ is then updated via a semi-Lagrangian calculation.

The cell-integrated average ρ is advanced by the conservative relation

$$\rho^{n+1}_{i-1/2} = \rho^n_{i-1/2} - (g_i - g_{i-1}), \tag{4}$$

where g_i represents the flux across the cell boundary during Δt.

The slope of the interpolation function at the cell center $d^n_{i-1/2}$ remains as a free parameter to be determined. It is this parameter that provides us a way to modify the interpolation function for suppressing numerical oscillation. Follows are two examples to define a CIP-CSL3 type scheme using conventional slope limiters.

The CIP-CSL3_UNO scheme:

We call a scheme *CIP-CSL3_UNO* if a UNO reconstruction[3] is used to evaluate the derivative $d^n_{i-1/2}$.

The CIP-CSL3_CW scheme:

In this scheme we adopt the approximation suggested by Colella and Woodward[4].

3. Numerical tests

We consider a one dimensional linear problem with an initial square profile. The computed results after 1000 steps of the CIP-CSL3_UNO and CIP-CSL3_CW are plotted in Fig.1. As expected, constrained by the oscillation suppressing reconstructions, both the schemes give well regulated and oscillation-less solutions. All the schemes conserve exactly the total mass.

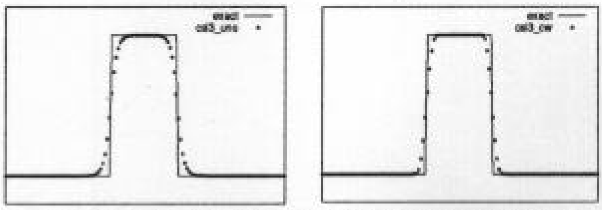

Fig.1 A transported square wave after 1000 step calculations with the CIP-CSL3_UNO scheme (left) and the CIP-CSL3_CW scheme (right).

We also computed the inviscid Burger's equation. A complicated initial condition same as that in [5] is specified.

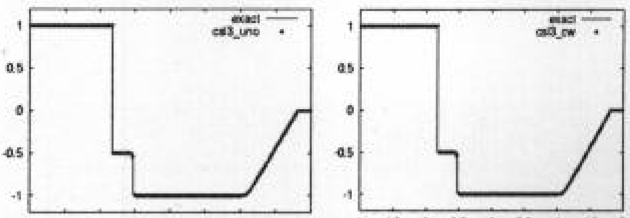

Fig.2 Solution of Burgers' equation at t=0.75 with an initial condition given in [5]. Displayed are the results computed by the CIP-CSL3_UNO scheme (left) and the CIP-CSL3_CW scheme (right).

The correct positions of the two shocks are obtained. The expansion waves are also accurately computed.

References

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