強い蒸発定常流の音速流限界

Strong evaporation and its approach to sonic steady state

大西善元（鳥取大・工・応用数理）

Yoshimoto Onishi, Tottori University, Tottori 680-8552

A steady evaporation flow never exceeds the local sonic speed however strong the evaporation process occurring at the condensed phase may be. This well-known feature can be well grasped by looking at the transition process of the strong transient evaporation flow toward its final state. Here this is shown by means of the numerical simulation of the flow based on the Boltzmann equation of BGK type subject to the diffusive boundary condition at the condensed phase(s). It will be shown that the expansion waves produced by the evaporation process at the interface surface, which are being swept backward by the supersonic flow, play an important role in the approach of the transient supersonic flow field to the sonic final state.

1. Introduction

An evaporation flow, when it attains the steady state, never exceeds the local sonic speed however strong the evaporation process occurring at the condensed phase may be (see, e.g., [1]). This is a well-known fact and several approximate analyses showing that the local flow Mach number is unity have been given ([2] and [3]). However, the physical explanation or reason why steady evaporation flows never exceed the sonic speed seems to be not well clarified. Here we try to show an aspect of this feature by means of the numerical simulation based on the Boltzmann equation of BGK type [4] subject to the diffusive boundary condition at the condensed phase(s). Actually we study the transient process of a strong evaporation flow, which is initially supersonic, and how it approaches the sonic state as it proceeds toward the steady state. It will be shown that the expansion waves produced by the evaporation process at the interface surface play an important role in the approach of the flow field to the sonic state. The expansion waves are propagating toward the condensed phase; however, owing to the supersonic evaporation flow prevailing, they are being swept backward by the flow, augmenting the thermodynamic quantities whereas reducing the velocity of the gas in the flow field. It may well state that it is these swept back expansion waves that lead the transient supersonic flow to its final sonic state.

2. Kinetic formulation

Let the condensed phase of a vapor be located at $x = 0$ and the half-space ($x > 0$) be occupied by the stationary vapor. Initially, the condensed phase and the vapor phase are in complete equilibrium at a temperature T_0 . The pressure, density and number density of the vapor at this state are P_0, ρ_0 and N_0 , respectively. Suppose that, at time $t = 0$, the temperature of the condensed phase is suddenly changed, i.e., $T_0 \to T_W$. This leads to the onset of phase change processes at the condensed phase, giving then rise to transient motions of the vapor. The Boltzmann equation of BGK type [4] for the description of the motions of the vapor may be written as

$$\frac{\partial f}{\partial t} + \xi_x \frac{\partial f}{\partial x} = N\nu_c\,(F_e - f) \tag{1}$$

$$F_e = \frac{N}{(2\pi RT)^{3/2}}\,\exp\Big\{-\frac{(\xi_x-u)^2+\xi_y^2+\xi_z^2}{2RT}\Big\} \tag{2}$$

$$\begin{bmatrix} N \\ Nu \\ \frac{3}{2}N\,k\,T \end{bmatrix} = \iiint \begin{bmatrix} 1 \\ \xi_x \\ \frac{1}{2}m[(\xi_x-u)^2+\xi_y^2+\xi_z^2] \end{bmatrix} \times f\,d\xi_x d\xi_y d\xi_z \tag{3}$$

$$P = N\,k\,T = \rho\,R\,T, \tag{4}$$

where t is the time; x is the coordinate; (ξ_x, ξ_y, ξ_z) is the molecular velocity vector; f is the molecular velocity distribution function, F_e being the local Maxwellian distribution characterized by the local fluid dynamic quantities; N, u, T , P and ρ are, respectively, the number density, the velocity, the temperature, the pressure and the density of the gas; m is the molecular mass; k is the Boltzmann constant and $R = k/m$ is the gas constant per unit mass of gas. ν_c is a constant associated with the collision frequency ($N\nu_c$ is the local collision frequency) and, hence, can be calculated either from the viscosity μ or from the thermal conductivity λ of the gas at a certain uniform state, say, at the initial uniform state, by the relation

$$N_0\nu_c = \frac{P_0}{\mu_0} = \frac{5}{2}R\,\frac{P_0}{\lambda_0}, \tag{5}$$

the suffix 0 being understood to indicate the quantities associated with the initial state. It may be noted here that, in the BGK model equation, the relation $\lambda = (5/2)R\,\mu$ holds and, hence, the Prandtl number is unity for this model equation.

The initial condition for the distribution function f for the present problem is, at $t = 0$

$$f = \frac{N_0}{(2\pi RT_0)^{3/2}}\,\exp\Big\{-\frac{\xi_x^2+\xi_y^2+\xi_z^2}{2RT_0}\Big\} \tag{6}$$

everywhere in the gas phase ($x > 0$) for all possible values of the molecular velocity vector (ξ_x, ξ_y, ξ_z). This is the stationary Maxwellian distribution function corresponding to the initial uniform state of the flow field. The boundary condition for f at the condensed phase at $x = 0$, on the other hand, may be specified as

$$f = \frac{N_W}{(2\pi RT_W)^{3/2}}\,\exp\Big\{-\frac{\xi_x^2+\xi_y^2+\xi_z^2}{2RT_W}\Big\} \tag{7}$$

for molecules leaving the surface of the condensed phase ($\xi_x > 0$), where N_W is the number density for molecules leaving the surface and is a unique function of the temperature T_W. Here, N_W is taken as the saturated vapor number density at the temperature T_W, which is determined by the Clapeyron-Clausius relation (see e.g., [5]) as

$$N_W = \frac{P_W}{k\,T_W}, \qquad \frac{P_W}{P_0} = \exp\left\{-\Gamma\left(\frac{T_0}{T_W}-1\right)\right\} \tag{8}$$

where P_W is the saturated vapor pressure at the temperature T_W and Γ is a non-dimensional parameter associated with the latent heat for vaporization per unit mass, h_L, and is defined by

$$\Gamma \equiv \frac{h_L}{RT_0}. \tag{9}$$

For the actual numerical calculation, the following new variables g and h, which are functions of t, x and ξ_x

$$\left[\begin{array}{c} g \\ h \end{array}\right] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[\begin{array}{c} 1 \\ \xi_y^2+\xi_z^2 \end{array}\right] f\, d\xi_y d\xi_z \tag{10}$$

have been introduced [6] in order to reduce the number of the independent variables from 5 to 3. With these new dependent variables g and h, the governing kinetic equations and the initial and boundary conditions have been rewritten, and then solved numerically by a difference scheme.

3. Characteristic parameters

For the present problem, we introduce the length scale L and the time scale τ_0 taken as

$$\left.\begin{aligned} L &\equiv \frac{\sqrt{\pi}}{2} l_0 = \left(\frac{2}{\gamma}\right)^{1/2} c_0 \frac{\mu_0}{P_0} \\ \tau_0 &\equiv \frac{L}{(2RT_0)^{1/2}} = \left(\frac{\gamma}{2}\right)^{1/2}\frac{L}{c_0} = \frac{1}{N_0\nu_c} = \frac{\mu_0}{P_0} \end{aligned}\right\} \tag{11}$$

where γ is the specific heat ratio ($\gamma = 5/3$ here) and $c_0 \equiv (\gamma RT_0)^{1/2}$ is the sound speed at the initial state. l_0 is the mean free path of the gas molecules at the initial state defined by

$$l_0 = \frac{(8RT_0/\pi)^{1/2}}{N_0\nu_c} = \frac{\mu_0}{P_0}\left(\frac{8RT_0}{\pi}\right)^{1/2} \tag{12}$$

It may be mentioned that, since the length scale L is of the order of the molecular mean free path l_0, the time scale τ_0 adopted here represents the mean collision time of gas molecules at the uniform initial state. With the fluid dynamic quantities at the initial state together with these length and time scales, the system of the governing kinetic equations and the initial and boundary conditions is appropriately nondimensionalized, giving the following nondimensional parameters

$$\frac{T_W}{T_0}, \qquad \Gamma \quad \left(\text{or } \frac{P_W}{P_0}\right) \tag{13}$$

which characterize the present flow field.

4. Results and discussion

A great number of cases have been calculated so far that have produced strong evaporation flows, which are supersonic initially and then gradually approaching their final sonic states. One of the typical cases is shown in Figs. 1 and 2, where a strong transient evaporation flow, which is supersonic in the bulk of the flow field, is taking place. The incipient flow is accelerating from being subsonic within the Knudsen layer, which is formed in the close vicinity of the condensed phase, becoming sonic somewhere around its edge and then to supersonic outside the layer. The expansion waves, which can be noticed by the fan-like part of the distributions in the figures, are being continuously swept backward with time into the flow, decreasing the velocity and increasing the thermodynamic quantities such as pressure, density and temperature. The origin of the fan-like expansion waves seems to be almost at the outer edge of the Knudsen layer. The position of this origin indicates the sonic point of the present flow because one of the expansion fan waves (or maybe the front wave of the expansion fan) should be standing still at the point, remaining there until the steady state of the flow is established. During the course, the tail part of the expansion waves is gradually reducing the supersonic speed of the flow toward its sonic speed as it is being constantly swept backward by the flow. The time when the very tail of the expansion waves being swept backward suppresses the supersonic flow is the time when the sonic state is established all over the flow field. It may be noted that these swept back expansion waves never lead the flow field to the subsonic state; when the flow field becomes subsonic, these waves are no longer swept backward by the flow but proceed through the subsonic flow field toward the condensed phase of evaporation side and will be eventually absorbed by it. The establishment of the sonic steady state, however, takes an infinite time in the present half-space problem because of the long persistence of the swept back expansion waves in the flow field. Strictly speaking, therefore, the exact sonic steady state is never attained in this case; only the limiting state is possible. Incidentally, it may be noted that the velocity u/c_0 has a hump (not the one due to the numerical errors). This is a structure that the velocity field has within the contact region, which has been noted and analyzed by Onishi et al. (see [7]).

Figure 3 shows in $x-t$ diagram a schematical view of an early stage of a supersonic flow of this kind due to the strong evaporation process taking place at the condensed phase. This may serve to get a clear idea of the flow pattern. A shock wave produced by the evaporation is propagating through a stationary gas, followed by a contact region (contact surface in Euler terms) behind. The expansion waves in a form of a fan produced at the same time are being swept backward into the flow field owing to the supersonic state. A schematical view of the distributions of the pertinent fluid dynamic quantities at a certain elapse of time are also shown in this flow state. Within the swept back expansion region near the boundary, the velocity has a straight line with a slope, the end point at the boundary being the sonic point. As the swept back expansion region expand with time, the slope becomes smaller and smaller until it gets totally flattened. The velocity in this final state is sonic all over.

This feature will also be looked at if we consider the two-surface problem in which another condensed phase is placed in parallel at a certain distance L from the first one. The temperatures of the first and second condensed phases are changed from the initial temperature T_0 and kept at T_{W1} and T_{W2}, respectively. In this two-surface problem, the nondimensional parameters characterizing the flow are as follows:

$$\frac{T_{W1}}{T_0}, \qquad \frac{T_{W2}}{T_0}, \qquad \Gamma \qquad Kn \equiv \frac{l_0}{L} \tag{14}$$

where Kn is the Knudsen number, which appears explicitly in this case. In Figs. 4 and 5, the variations of the local flow Mach number u/c with time t/τ_0 are shown. τ_0 is defined by $\tau_0 = L/(2RT_0)^{1/2}$ in this case. In both cases, the evaporation flow is supersonic at early stages. The difference between the two cases, however, manifest itself right after the initial shock wave has interacted with the condensed phase at $x = L$. The interaction causes the condensation at the condensed phase; the mass flow behind the shock wave will condense onto the condensed phase. In some cases, however, a part of the mass flow remains unabsorbed by it and causes in its close vicinity the compression region, which starts propagating as a reflected shock wave toward the evaporation side at $x = 0$. This situation is clearly visible in Fig. 4, where the swept back expansion waves, the reflected shock wave and their interaction itself (see e.g., the time $t/\tau_0 = 6.0$ and its after) are all lowering the flow velocity from the initial

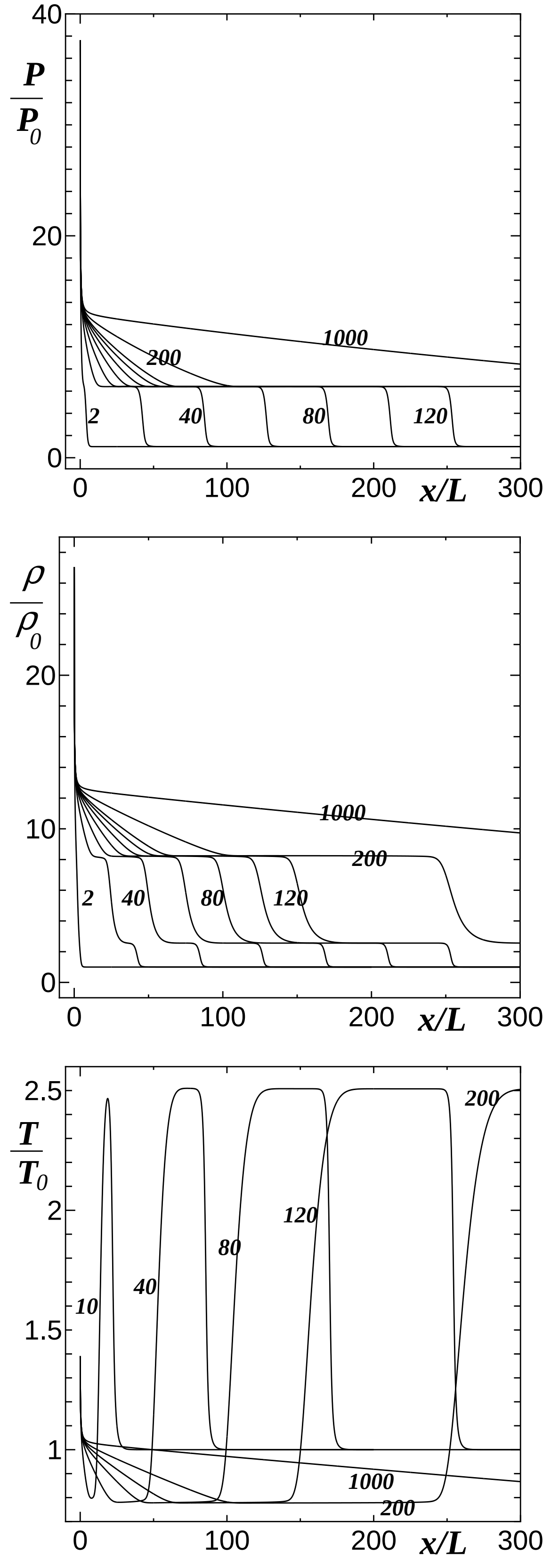

Fig. 1: Transient distributions of the pressure, density and temperature for $T_W/T_0 = 1.6$, $\Gamma = 11.0$ ($P_W/P_0 = 61.86781$ and $N_W/N_0 = 38.66738$). The interface surface is at $x = 0$. The numbers in the figures indicate the time t/τ_0.

supersonic speed to the final subsonic one. This subsonic state is in this case the steady state achieved.

In the case of Fig. 5, on the other hand, owing to the strong evaporation process and the consequent strong ini-

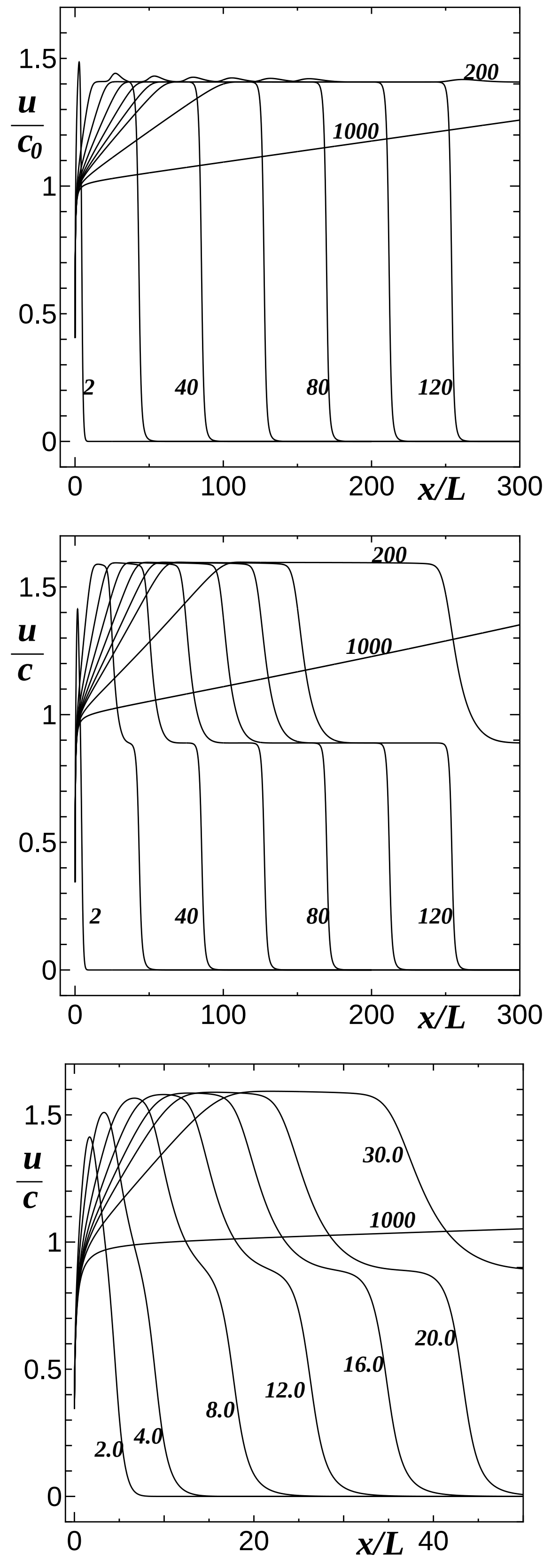

Fig. 2: Transient distributions of the velocity and the local Mach number for $T_W/T_0 = 1.6$, $\Gamma = 11.0$ ($P_W/P_0 = 61.86781$ and $N_W/N_0 = 38.66738$). c is the local sound speed. The interface surface is at $x = 0$. The numbers in the figures indicate the time t/τ_0. Note that the velocity u/c_0 has a hump structure within the contact region [7], not the numerical errors.

tial shock wave, the gas pressure behind the shock wave becomes sufficiently high enough to push the whole mass flow

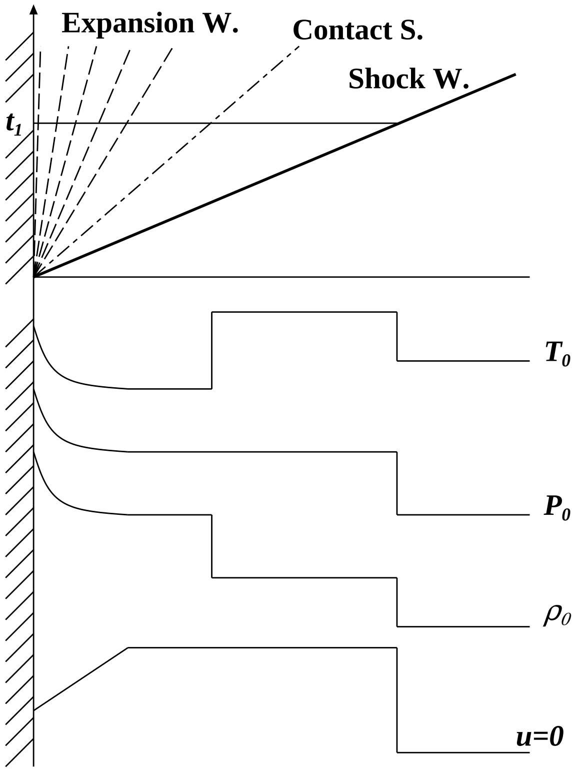

Fig. 3: A schematic view of an early stage of the propagation of waves, a shock wave, a contact surface and swept-back expansion waves, in the transient supersonic flow field caused from the onset of strong evaporation process at the condensed phase. The profiles of the fluid dynamic quantities at a certain time t_1 are shown below. The flow velocity u is supersonic ($u > c$) except at the boundary surface where the flow is sonic ($u = c$). Note that the curved parts of the profiles of the thermodynamic quantities in the swept back expansion region are more or less exaggerated.

into the condensation surface (at $x = L$) after the shock wave has reached it (see the mass flow in Fig. 6). The swept back expansion waves continue to augment the gas pressure as they are propagating backward toward the condensation surface, reducing the flow velocity from supersonic toward sonic. During the course of time, the whole mass flow is being condensing onto the condensation surface due to the existing large difference between the gas pressure formed near there and the saturated vapor pressure corresponding to the temperature of the surface itself. This situation continues until the flow velocity becomes sonic all over the flow field except the Knudsen layers. However, owing to the nature of the swept back expansion waves, this flow field seems never to attain the exact steady sonic state; only its limiting state may be possible just as in the case of the half-space problem. Or at least, it takes an extremely long time for the flow field to attain its steady state. The reason is that, as the flow field approches the sonic state, the swept back expansion waves become hardly mobile and persist in the flow field until they diffuse away completely because of the viscous action. The expansion waves at this stage, however, are now extremely weak and may be considered to be hardly subject to the viscous action. Therefore, virtually immobile expansion waves prevail in the bulk of the present flow field, the state of which is not the steady state strictly speaking, because the steady state is, presumably, the state at which these expansion waves have completely disappeared. Figure 7 shows this nature of persistence of the expansion waves in the flow field in terms of the local flow Mach number near the condensation surface. Slowly varying nature of the local flow Mach number can be seen owing to the almost immobile swept back expansion waves prevailing in the flow field.

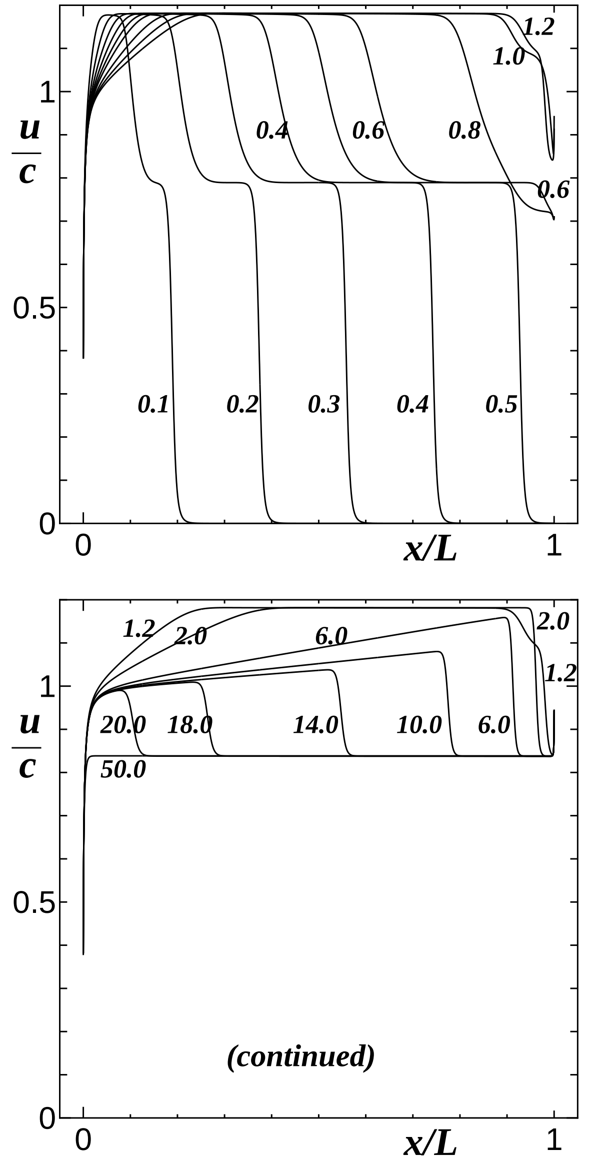

Fig. 4: Transient distributions of the local flow Mach number u/c of a vapor between the two parallel plane condensed phases placed at a distance L apart. $T_{W1}/T_0 = 1.6$, $T_{W2}/T_0 = 1.0$, $\Gamma = 9.0$ ($P_{W1}/P_0 = 29.22428$ and $N_{W1}/N_0 = 18.26518$) and $Kn = 0.005$. T_{W1} and T_{W2} are the temperatures of the condensed phases at $x = 0$ and $x = L$, respectively. The numbers in the figures indicate the time t/τ_0, τ_0 being defined in this case by $\tau_0 = L/(2RT_0)^{1/2}$. Incidentally, the values of u/c at time $t/\tau_0 = 50.0$ are $u/c = 0.3771295$ at $x/L = 0.0$, $u/c = 0.8385715$ at $x/L = 0.5$ and $u/c = 0.9458901$ at $x/L = 1.0$. The Mach number of the initially produced shock wave is about $M_s = 1.865$.

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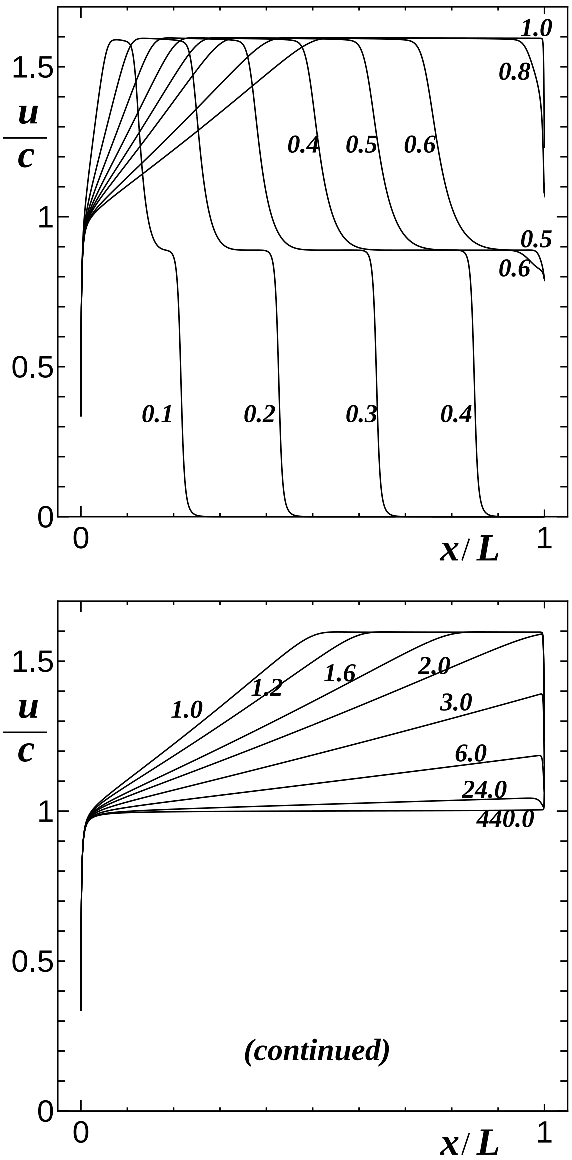

Fig. 5: Transient distributions of the local flow Mach number u/c of a vapor between the two parallel plane condensed phases placed at a distance L apart. $T_{W1}/T_0 = 1.6$, $T_{W2}/T_0 = 1.0$, $\Gamma = 11.0$ ($P_{W1}/P_0 = 61.86781$ and $N_{W1}/N_0 = 38.66738$) and $Kn = 0.005$. T_{W1} and T_{W2} are the temperatures of the condensed phases at $x = 0$ and $x = L$, respectively. The numbers in the figures indicate the time t/τ_0, τ_0 being defined in this case by $\tau_0 = L/(2RT_0)^{1/2}$. Incidentally, the values of u/c at time $t/\tau_0 = 440.0$ are $u/c = 0.3343818$ at $x/L = 0.0$, $u/c = 1.000155$ at $x/L = 0.5$ and $u/c = 1.143063$ at $x/L = 1.0$. The Mach number of the initially produced shock wave is about $M_s = 2.313$.

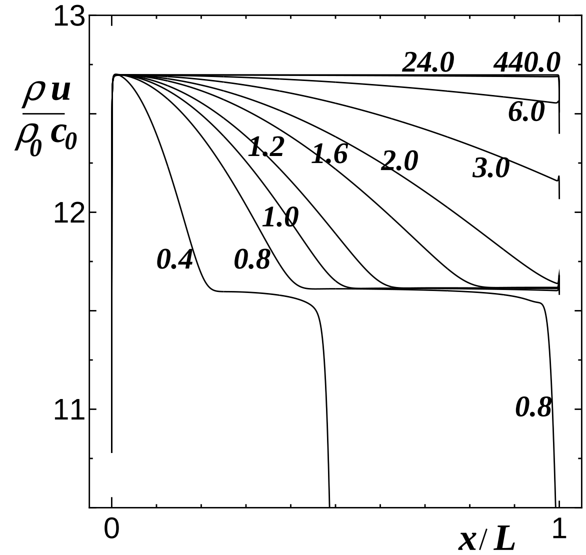

Fig. 6: Transient distributions of the mass flow $\rho u/(\rho_0 c_0)$ of a vapor between the two parallel plane condensed phases. $T_{W1}/T_0 = 1.6$, $T_{W2}/T_0 = 1.0$, $\Gamma = 11.0$ ($P_{W1}/P_0 = 61.86781$ and $N_{W1}/N_0 = 38.66738$) and $Kn = 0.005$. The numbers in the figures indicate the time t/τ_0.

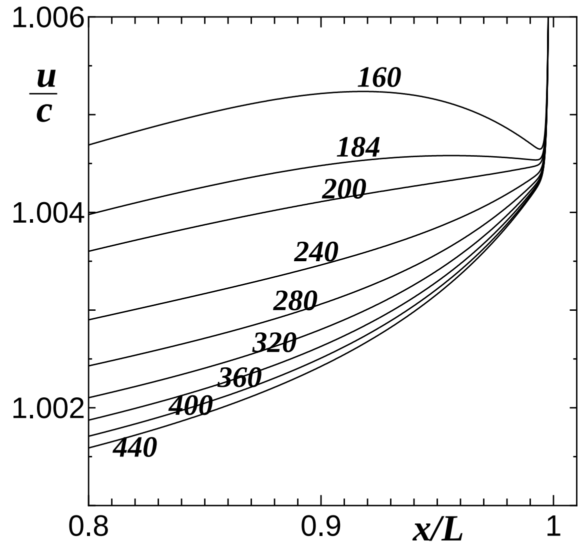

Fig. 7: A portion of Fig. 5, which shows the time development of the local flow Mach number u/c near the condensation surface at $x = L$, the evaporation surface being at $x = 0$. The flow field is at its final stage. $T_{W1}/T_0 = 1.6$, $T_{W2}/T_0 = 1.0$, $\Gamma = 11.0$ ($P_{W1}/P_0 = 61.86781$ and $N_{W1}/N_0 = 38.66738$) and $Kn = 0.005$. The numbers in the figure indicate the time t/τ_0.