Nonlocal Electron Thermal Conduction in Laser Implosions

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A 1-D electron-thermal-conduction calculation code based on the Vlasov-Fokker-Planck equation has been developed. Our code includes the electron-ion and electron-electron collisions. The electron distribution function is expanded into the zeroth-, first-, and second-order terms of Legendre polynomials, which are solved simultaneously. We combined our code with the 1-D hydrodynamic code to calculate the nonlocal thermal conduction in the ICF implosion.

The inertial confinement fusion (ICF) plasma has wide parameter ranges where the electron number density changes from 10^{19} to $10^{26} cm^{-3}$, and the electron temperature varies from room temperature to $10^5 eV$. In the case of lower density and higher temperature, the electron-ion and electron-electron collision decrease and the electron mean free path can be similar to the temperature gradient scale length. In such cases, the electron velocity distribution function diverges from Maxwellian, and the Spitzer-Härm model[1] overestimates the thermal flux beyond the physical limit. In order to remedy this overestimation, the flux-limiter was introduced to the SH model, and the electron thermal flux is given by Eq.(1),

$$q_e = min(fq_{FS}, q_{SH}) \qquad (1)$$

where q_e is the electron thermal flux, q_{FS} is the free-streaming thermal flux defined as $q_{FS} = n_e T_e (T_e/m_e)^{\frac{1}{2}}$, n_e is the electron number density, T_e the electron temperature in energy unit, and m_e the electron mass, respectively. This is called the flux-limited Spitzer-Härm model[2]. This model gives a reasonable flux, and has been widely used for many simulations. However, this model is empirical, since it requires the flux-limiter to be determined from the comparison with experimental results and also from simulations. For the general conditions covering all the parameter region of the ICF plasma, we should calculate the electron thermal conduction based on the Vlasov-Fokker-Planck (VFP) equation[3], which describes the time evolution of the electron velocity distribution function.

We have developed our VFP code. We adopted the moving coordinate attached to the moving ion as the frame of the electron distribution function. The electron velocity distribution function f is expanded by the Legendre function to the zero-th f_0, first f_1 and second f_2 order parts[4] that are solved simultaneously. We used the collision operators for electron-ion collision[4] and

electron-electron collisions[5]. The electric field is calculated in the quasi-charge neutral condition[6]. Also, we introduce the friction term against the change of temperature in order to introduce the effect of the real heat capacity resulting from the general equation of state. Our Vlasov-Fokker-Planck (VFP) code aimes at connecting to a hydrodynamic code. Therefore, this code is used as one of the subroutines of the hydrodynamic code.

In our code, we used the operator splitting method for the electron-electron collision and the electron-ion collision;

$$\frac{f_s^* - f_s^n}{\Delta t} = C_{ee} + S_0 \qquad (2)$$

$$\frac{f_s^{n+1} - f_s^n}{\Delta t} = C_{ei} + \frac{(f_s^* - f_s^n)}{\Delta t} \qquad (3)$$

where, C_{ee} and C_{ei} are the electron-electron collision term and the electron-ion collision term, respectively. f_s^n $(s = 0, 1, 2)$ is the distribution function at time step n, and f_s^* is defined as the intermediate value. f_s^{n+1} is defined at time step $n + 1$. S_0 is the source term. The electron velocity distribution function is expanded up to $\ell = 3$ mode by Legendre polynomials [4] as shown below.

Zero-th order;

$$
\begin{aligned}
\frac{\partial f_0}{\partial t} &+ \frac{1}{3}\left\{ \frac{v}{\eta}\frac{1}{x^2}\frac{\partial}{\partial x}(x^2 f_1) - \frac{a}{v^2}\frac{\partial}{\partial v}\left(v^2 f_1\right) \right\} \\
&- \frac{2}{15v^2}\frac{\partial U}{\partial x}\frac{\partial}{\partial v}\left(v^3 f_2\right) \\
&= C_{ee}(f_0, f_0) + S_0
\end{aligned}
$$

$$(4)$$

First order;

$$\frac{\partial f_1}{\partial t} + v\frac{\partial f_0}{\partial x} - a\frac{\partial f_0}{\partial v}$$

$$+ \frac{2}{5}\left\{\frac{v}{x^2}\frac{\partial}{\partial x}(x^2 f_2) - \frac{a}{v^3}\frac{\partial}{\partial v}\left(v^3 f_2\right)\right\}$$

$$- \frac{\partial U}{\partial x}\cdot f_1$$

$$- \left\{\frac{2}{5}\frac{\partial U}{\partial x} + \frac{1}{5x^2}\frac{\partial}{\partial x}(x^2 U)\right\}v^2\frac{\partial}{\partial v}\left(\frac{f_1}{v}\right)$$

$$= -\nu_{ei}f_1 \qquad (5)$$

Second order;

$$\frac{\partial f_2}{\partial t} + v\frac{\partial f_1}{\partial x}$$

$$- \frac{v}{3x^2}\frac{\partial}{\partial x}(x^2 f_1) - \frac{2}{3}av\frac{\partial}{\partial v}\left(\frac{f_1}{v}\right)$$

$$+ \frac{3}{7}\left\{v\frac{\partial f_3}{\partial x} - \frac{a}{v^4}\frac{\partial}{\partial v}\left(v^4 f_3\right)\right\}$$

$$+ \frac{1}{7}\left\{\frac{2}{x^2}\frac{\partial(x^2 U)}{\partial x} - 4\frac{\partial U}{\partial x}\right\}f_2$$

$$- \left\{\frac{\partial U}{\partial x} - \frac{1}{3x^2}\frac{\partial(x^2 U)}{\partial x}\right\}v\frac{\partial f_0}{\partial v}$$

$$- \frac{1}{21}\left\{8\frac{\partial U}{\partial x} + \frac{3}{x^2}\frac{\partial(x^2 U)}{\partial x}\right\}v\frac{\partial f_2}{\partial v}$$

$$= -3\nu_{ei}f_2 \qquad (6)$$

Third order;

$$\frac{\partial f_3}{\partial t} + \frac{3}{5}\left\{v\frac{\partial f_2}{\partial x} - av^2\frac{\partial}{\partial v}\left(\frac{f_2}{v^2}\right)\right\} = -6\nu_{ei}f_3 \qquad (7)$$

where f_0, f_1, f_2 and f_3 are the isotropic part ($\ell = 0$), $\ell = 1$, $\ell = 2$ and $\ell = 3$ mode, respectively. For the closure of these coupled equations, we use the simplified f_3 equation without the correction terms of the hydrodynamic motion. Also, some additional terms for the non-planar geometry are omitted, and the initial condition of f_3 is reset to 0 at every time step.

In the above equations, C_{ee} considers interaction among f_0 itself only. ν_{ei} is the electron-ion collision frequency given by $\nu_{ei} = \phi 4\pi n_e Z^*\left(\frac{e^2}{m_e}\right)^2 ln\Lambda/v^3$, which effectively includes the corrections of higher order terms[7] neglected in the electron-electron collision operator. ϕ is given by $\phi = (Z^* + 4.2)/(Z^* + 0.24)$. The effective charge Z^* is defined by $Z^* = <Z^2>/<Z>$, where $<Z^2>$, $<Z>$ are the square charge and charge averaged over the ion species, respectively. e is the electron charge, and $ln\Lambda$ Coulomb logarithm[8]. U is the ion velocity.

S_0 and η are a source term and a friction term needed to consistently couple with a hydrodynamic code. S_0 accounts for the change in the Maxwell distribution function $f_M = n_e(m_e/2\pi T_e)^{\frac{3}{2}}exp(-m_e v^2/2T_e)$, i.e., $S_0 = \delta(f_M)$, due to changes in the electron density and temperature from the ionization, radiation transport, laser absorption, and pdV work calculated by the hydrodynamic code before the FP calculation. The detail definition of S_0 will be described later. The friction

term η was introduced to effectively take into account the heat capacity for the real gas. This term restrains the temperature change due to the thermal conduction according to the real gas heat capacity per electron c_{ve}. The ideal gas capacity per electron c_{ve} is $\frac{3}{2}$. For real gas, η is given by $\eta = 2\,c_{ve}/3$, where c_{ve} is obtained from the hydrodynamic code. .

For electron-ion collision calculation, we used the MacCormak scheme[9] in the velocity direction. The equations for f_0, f_1, and f_2 above are solved using a tri-diagonal solver. These equations are solved from $j = J(= jmax)$ to $j = 1$ in a reversed order. Therefore, when we calculate at j, the $j+1$ terms have been already solved, and we can calculate the j order terms with the backward differencing term of $\frac{f_j - f_{j-1}}{\Delta v_{j-\frac{1}{2}}}$ and the forward differencing term of $\frac{f_{j+1} - f_j}{\Delta v_{j+\frac{1}{2}}}$. In the calculation of f_j, both terms f_{j+1}^{n+1} and f_{j-1}^n, required for the calculation, are already known, and we can solve the flux in the velocity direction without solving the matrix. It is a merit of using the MacCormak sheme in the velcoity direction. This enable us to calculate the flux in the both direction of the configuration space coordinate and the velocity space coordinate simultaneously, contribute to the numerical stability and save the computation time. For the configuration space, we adopted the full implicit finite difference method with the staggered grid shown in Fig.1.
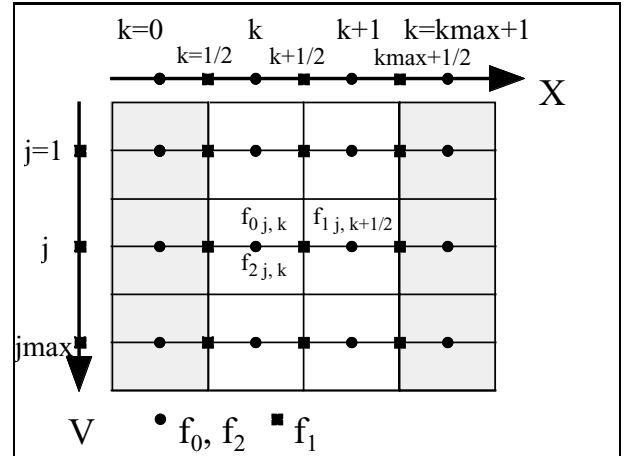

Fig. 1: Calculation grid configuration

In calculating of the electric field a ($=\frac{eE}{m_e}$), we solve the Poisson equation using an implicit moment method[11] with current free condition, and the electric field a in the finite difference form is given by

$$a_{k+\frac{1}{2}}^{n+1} = \frac{numerator}{denominator} \qquad (8)$$

where

$$numerator =$$

$$- \sum_{j=1}^{J}\frac{1}{\alpha_j}f_{1\,j,k+\frac{1}{2}}^n v_j^3\Delta v_j$$

$$+ \Delta t\sum_{j=1}^{J}\frac{1}{\alpha_j}\left\{\frac{f_{0\,k+1} - f_{0\,k}}{\Delta x_{k+\frac{1}{2}}} + \frac{2}{5}\left(\frac{f_{2\,k+1} - f_{2\,k}}{\Delta x_{k+\frac{1}{2}}}\right)\right\}_j^{**}$$

$$v_j^4 \Delta v_j \Delta t$$

$$- \quad \Delta t \sum_{j=1}^{J} \frac{1}{\alpha_j} \left\{ \frac{2}{5} (\nabla U)_{k+\frac{1}{2}} + \frac{1}{5} (\nabla \cdot U)_{k+\frac{1}{2}} \right\}$$

$$\times \quad \left(\frac{f_{1\,j+1} - f_{1\,j}}{\Delta v_j} \right)_{k+\frac{1}{2}}^{n+1} v_j^4 \Delta v_j \times \theta_0$$

$$- \quad \Delta t \sum_{j=1}^{J} \frac{1}{\alpha_j} \left\{ \frac{2}{5} (\nabla U)_{k+\frac{1}{2}} + \frac{1}{5} (\nabla \cdot U)_{k+\frac{1}{2}} \right\}$$

$$\times \quad \left(\frac{f_{1\,j+1} - f_{1\,j}}{\Delta v_j} \right)_{k+\frac{1}{2}}^{n} v_j^4 \Delta v_j \times (1 - \theta_0) \qquad (9)$$

$$denominator =$$

$$- \quad \Delta t \sum_{j=1}^{J} \frac{v_j^3 \Delta v_j}{\alpha_j} \theta_0$$

$$\times \quad \left\{ \frac{f_{0\,j+1} - f_{0\,j}}{\Delta v_{j+\frac{1}{2}}} + \frac{2}{5 v_j^3} \left(\frac{v_{j+1}^3 f_{2\,j+1} - v_j^3 f_{2\,j}}{\Delta v_{j+\frac{1}{2}}} \right) \right\}_{k+\frac{1}{2}}^{n+1}$$

$$- \quad \Delta t \sum_{j=1}^{J} \frac{v_j^3 \Delta v_j}{\alpha_j} (1 - \theta_0)$$

$$\times \quad \left\{ \frac{f_{0\,j} - f_{0\,j-1}}{\Delta v_{j-\frac{1}{2}}} + \frac{2}{5 v_j^3} \left(\frac{v_j^3 f_{2\,j} - v_{j-1}^3 f_{2\,j-1}}{\Delta v_{j-\frac{1}{2}}} \right) \right\}_{k+\frac{1}{2}}^{n} \qquad (10)$$

In Eqs.(9) and (10), θ is the parameter of the ratio of explicit and implicit fluxes in the MacCormak scheme. α is given by

$$\alpha_j = 1 + \nu_{ei}^* \Delta t - \frac{3}{5} (\nabla U)_{k+\frac{1}{2}} \Delta t + \frac{1}{5} (\nabla \cdot U)_{k+\frac{1}{2}} \Delta t.$$

Furthermore, we replace the values defined at time step $n+1$ by the predicted values given by

$$f_{0\,j,k}^{n+1} \quad \approx \quad f_{0\,j,k}^{n} + \Delta S_{0,j,k} \qquad (11)$$

$$f_{2\,j,k}^{n+1} \quad \approx \quad f_{2\,j,k}^{n} \qquad (12)$$

Here, we approximate f_0^{n+1} by $f_0^n + S_0$. $a^{n+\frac{1}{2}}$ defined at time step $n + \frac{1}{2}$ is given as the average of these a^n and a^{n+1}

$$a_{k+\frac{1}{2}}^{n+\frac{1}{2}} \quad = \quad \frac{a_{k+\frac{1}{2}}^{n+1} + a_{k+\frac{1}{2}}^{n}}{2} \qquad (13)$$

In actual calculations, we use the normalized electric field $a_{k+\frac{1}{2}}^{n+\frac{1}{2}}$ instead of $a_{k+\frac{1}{2}}^{n}$ and $a_{k+\frac{1}{2}}^{n+1}$ for the sake of simplicity.

In order to study the nonlocal electron transport effects on the hydrodynamics, this VFP code should be connected with a hydrodynamic code consistently. We replace the thermal conduction routine based on the Spitzer-Härm model by the VFP code, where the VFP routine is operated as the one of the subroutines in the hydrodynamic code. When the VFP routine is called

from the hydrodynamic calculation routine, some physical variables are transfered into the VFP routine.

From the hydrodynamic code, those parameters listed bellow are transfered at every time step.

- Configuration mesh location x

- The elecron number density n_e

- The elecron temperature T_e

- The ionization degree $< Z >$ and average value of $< Z^2 >$

- The ion velocity U

- The mass density change of ρ^n and ρ^{n-1}

- The source term S_0

- The electron heat capacity c_{ve}

For the source term of the VFP routine, we assume a Maxwellian distribution. Therefore, the change of the temperature and the electron density due to the ionization, radiation transport, laser absorption, and hydrodynamic motion are included via changes of the Maxwell distribution function. We estimate the source term in two different forms. For the change of temperaure with constant density, we use the source term given by

$$\Delta f_0(v)_a = f_M(n_e, T_b) - f_M(n_e, T_a) \qquad (14)$$

This temperature change includes processes such as the radiation transport and laser aborption. For changes of the electron number density due to ionization at constant temperaure we use

$$\Delta f_0(v)_b = f_M(\Delta n_e, T_a) \qquad (15)$$

Then, the source term S_0 is given by the sum of the Eqs.(14) and (15) as;

$$S_0 = \Delta f_0(v)_a + \Delta f_0(v)_b \qquad (16)$$

Next, we describe the method of determining the electron temperature using the VFP routine. We treat the source term in the hydrodynamic calculation to be the Maxwell distribution function. However, in the VFP routine, we do not assume the distribution function to be Maxwellian. Therefore, the distribution function is independently preserved in the VFP routine. We input the contribution of other processes as a source term into the VFP routine, and the electron temperature is returned to the hydrodynamic code after the electron thermal conduction calculation. At that time, the distribution function should be consistent with the density and temperature of the hydrodynamic calculation. Therefore, after the calculation in the VFP routine is done, we return the effective temperature, T_{eff}, calculated from the non-Maxwellian distribution function back to the hydrodynamic code, where T_{eff} is given by

$$T_{eff} = \frac{4\pi m_e}{3 n_e} \int_0^\infty v^4 f_0 dv. \qquad (17)$$

From the point of view of the hydrodynamic code, we might say that the VFP thermal conduction routine is a part of the energy equation. We describe the hydrodynamic equation in the Lagrangian form given by

$$\frac{d\rho}{dt} = \rho\nabla \cdot \vec{u} \tag{18}$$

$$\rho\frac{du}{dt} = -\nabla(P + Q) \tag{19}$$

$$\rho\frac{d\epsilon_i}{dt} = -(P_i + Q)\nabla \cdot \vec{u} - \nabla \cdot \vec{q_i} + Q_{ei} \tag{20}$$

$$\rho\frac{d\epsilon_e}{dt} = -P_e\nabla \cdot \vec{u} - \nabla \cdot \vec{q_e} - Q_{ei} + S_L + S_r \tag{21}$$

where, ρ is the mass density, u is the fluid velocity, p_i and p_e are the ion pressure and the electron pressure, respectively. $p = p_e + p_i$ is the total pressure, Q the artficial viscosity, T_e and T_i are the electron and ion temperature, respectively. ε_e and ε_i are the specific internal energies, respectively, Q_{ei} the electron-ion energy relaxation term and S_L and S_r are, respectively, the laser absorption term and the energy deposition due to the radiation transport.

In the calculation of the energy equation of Eqs.(20) and (21), the equations are usually expressed in terms of the temperature. Using the ion and electron specific heat c_i, c_e, these equations are reduced to

$$\rho c_i\frac{dT_i}{dt} = -(P_i + Q)\nabla \cdot \vec{u} - \nabla \cdot \vec{q_i} + Q_{ei} \tag{22}$$

$$\rho c_e\frac{dT_e}{dt} = -P_e\nabla \cdot \vec{u} - \nabla \cdot \vec{q_e} - Q_{ei} + S_L + S_r \tag{23}$$

where T_i and T_e are the ion and electron temperature, respectively. When we calculate the electron energy equation using the VFP code, we divide the energy equation into two part using the operator splitting method as;

$$\rho c_e\frac{T_e^* - T_e^n}{dt} = -P_e\nabla \cdot \vec{u} - Q_{ei} + S_L + S_r \tag{24}$$

$$\rho c_e\frac{T_e^{n+1} - T_e^n}{dt} = -\nabla \cdot \vec{q_e} + \rho c_e\frac{T_e^* - T_e^n}{dt} \tag{25}$$

where T_e^n is the electron temperature defined at time step n, and T_e^* is the intermediate value. T_e^{n+1} is the electron temperature defined at the time step $n+1$. We regard the contribution due to $T_e^* - T_e^n$ as the source term of the distribution function of f_0. Finally, by replacing Eq.(23) by the VFP calculation routine, we can solve the energy equation with the VFP routine and obtain the electron temperature T^{n+1}, which is the electron effective temperaure T_{eff} in the VFP routine. Equations (22) and (23) are equivalent to Eqs. (2) and (3).

In order to calculate the nonlocal thermal conduction and to study the effect in the ICF implosion, we combined our VFP code with the hydrodynamic code LILAC[10].

We simulated the implosion of a polystyrene CH shell of diameter $900\mu m$ and thickness $20\mu m$ filled with $15atm$ D_2 gas. The $0.35\mu m$ laser pulse was a 1-ns duration square with a rise time of $200ps$ and a constant power of $25TW$ from $0.4ns$ to $1.4ns$. The irradiation intensity on the target was $9 \times 10^{14}W/cm^2$. In Fig.2, we show the density and temperature profiles at 1.5ns. We can see that VFP calculation gives a slightly different profiles compared to that of the flux-limited SH calculation with flux-limiter of 0.06 and 0.07.
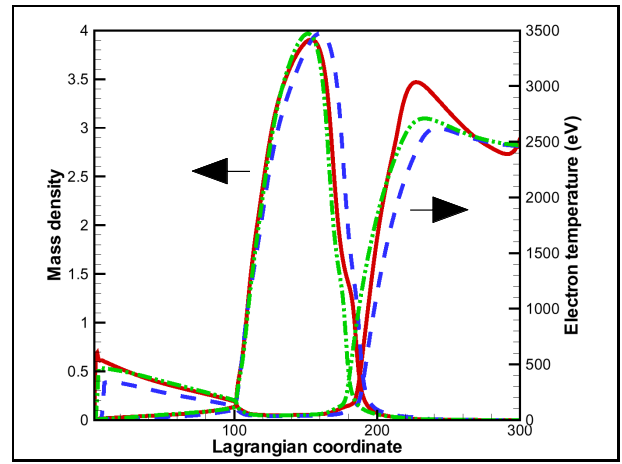

Fig. 2: Density and temperature profiles at 1.5ns. Solid line is FP. Dashed line shows flux-limited SH with flux-limiter of 0.07. Thin line show the case with flux-limiter of 0.06.

In conclusion, we have developed a 1-D electron-thermal-conduction calculation code based on the Vlasov-Fokker-Planck equation. We combined our code with the 1-D hydrodynamic code to calculate the nonlocal thermal conduction in the ICF implosion. We successfully calculated CH target implosions. VFP calculation gives density and temperature profiles that are slightly different from that of flux-limited SH calculation.

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