A Preconditioned Numerical Method for Gas-Liquid Two-Phase Flows

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A preconditioned numerical method to solve gas-liquid two-phase flows is proposed. The present method employs a finite-difference MacCormack method combined with TVD scheme, and a homogeneous equilibrium cavitation model. The method proposed here permits simple treatment of the whole gas-liquid two-phase flow field including wave propagation, large density changes and incompressible flow characteristics at low Mach number. By this method, a two-dimensional decelerating cascade and channel flow was computed. Numerical results such as some data related to performance characteristics of hydrofoils are provided.

1. Introduction

Gas-liquid two-phase flows such as cavitating flow are characterized by mixed incompressible/compressible nature with large variations of the local Mach number. Indeed, the speed of sound with a large range of magnitude can coexist in the two-phase mixture. For simulation of such flows, therefore, a numerical method which can handle accurately any Mach number is necessary. For such flow, compressible flow model with preconditioning method [1,2] is advantageous.

On the other hand, cavity flow is possible to model into an apparent single-phase flow by using a concept of the homogeneous equilibrium model. Under this model concept, the pressure for gas-liquid two-phase media is determined by using a combination of two equations of state for gas phase and liquid phase written as [3]:

$$\rho = \frac{p(p+p_c)}{K(1-Y)p(T+T_c)+RY(p+p_c)T} \tag{1}$$

where, ρ, p, Y, and T are the mixture density, pressure, mass fraction of the vapor and the temperature, respectively. The apparent compressibility is considered, and the sound speed c becomes

$$\frac{1}{c^2} = \frac{\partial \rho}{\partial T}(\rho C_p)^{-1} + \frac{\partial \rho}{\partial p} \tag{2}$$

C_p is the specific heat capacity at constant pressure.

2. Fundamental Equations

Based on above modeling concept and neglecting the surface tension, the 2-D preconditioned governing equations for the mixture mass, momentum, energy and the gas-phase mass can be written in the curvilinear coordinates as follows:

$$\Gamma^{-1}\frac{\partial \boldsymbol{W}}{\partial t} + \frac{\partial(\boldsymbol{E}-\boldsymbol{E}_v)}{\partial \xi} + \frac{\partial(\boldsymbol{F}-\boldsymbol{F}_v)}{\partial \eta} = \boldsymbol{R} \tag{3}$$

where $\boldsymbol{W}(=[p,u,v,T,Y]^T)$ is the unknown variable vector, $\boldsymbol{E}(=J[\rho U, \rho uU+\xi_x p, \rho vU+\xi_y p, \rho UH, \rho UY]^T)$ and $\boldsymbol{F}$ are the flux vectors and $\boldsymbol{E}_v(=J[0, \eta_x\tau_{xx}+\eta_y\tau_{xy}, \eta_x\tau_{yx}+\eta_y\tau_{yy}, \eta_x T_{11}+\eta_y T_{22}, \eta_x\Re Y_x+\eta_y \Re Y_y]^T)$ and $\boldsymbol{F}_v$ are the viscous terms presented by the stress tensor τ of the compressible Navier-Stokes flow. $\boldsymbol{R}$ is a source term for phase change and $\Re$ is effective exchange coefficient.

In this study, the preconditioning matrix Γ^{-1} is formed by the addition of the vector $\theta[1,u,v,H,Y]^T$ to the first column of the Jacobian matrix $\partial\boldsymbol{Q}/\partial\boldsymbol{W}$, where $\boldsymbol{Q}(=J[\rho,\rho u,\rho v,e,\rho Y]^T)$ is the conservative unknown variables.

In this paper, the governing equations (3) are numerically integrated by using the finite-difference MacCormack method. Also, a TVD scheme is applied to enhance the numerical stability, especially for the existence of steep gradient in density as well as pressure near the gas-liquid interface. Details will be refer to the Ref.[3,4].

3. Numerical Results

As an example of numerical results, Fig.1 shows a comparison of pressure distribution around hydrofoil for a two-phase flow at inlet Mach number of 0.02 and Reynolds number of 3×10^5. The solution obtained with the preconditioning is much more stable than that without the preconditioning. In the full paper, the effectiveness of the present preconditioning method, detailed observations of cavity flows and comparisons of predicted results with experiments are provided.

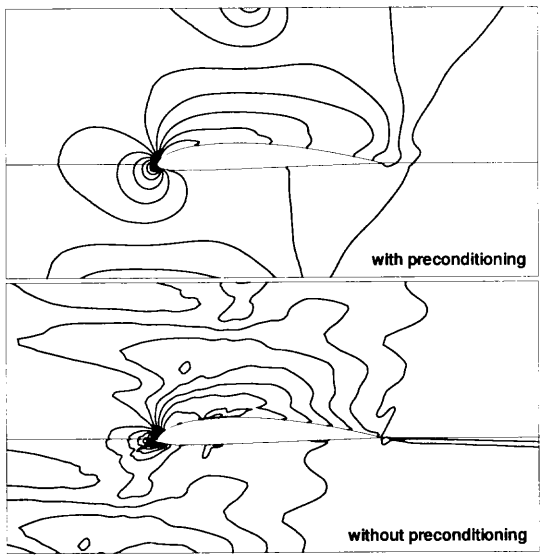

Fig.1: Pressure contours with preconditioning (top) and without preconditioning (bottom)

References

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