A semi-Lagrangian/semi-implicit model for atmospheric flows

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It's getting widely accepted that a numerical model with a spatial resolution finer than a few kilometers should include explicitly the effects of non-hydrostatics and compressibility. However, directly solving acoustic wave as well as gravity wave, which appear as the "fast modes" in atmospheric motion, restricts the time integration step to an impractically small value if a completely explicit time marching approach is used.

This work presents a compressible and non-hydrostatic model by using a two-step semi-Lagrangian/ semi-implicit formulation. Rather than the conventional terrain following mesh, a grid based on Cartesian coordinate is used to represent topography. Manipulations, such as the sub-grid flux evaluation and velocity modification to the surface cells, have been adopted to get a more realistic representation for topography.

The set of fully compressible and non-hydrostatic governing equations are similar to those in [1]. Semi-Lagrangian/semi-implicit formulations are used for all prognostic variables. For any evolution equation of dependent variable ϕ

$$\frac{d}{dt}\phi(\mathbf{x},t) = \mathcal{L}[\phi(\mathbf{x},t)], \qquad (1)$$

a time integration algorithm using two-step semi-Lagrangian advection can be written as

$$\frac{\phi(\hat{\mathbf{x}},t+\Delta t) - \phi(\hat{\mathbf{x}} - \vec{V}\Delta t, t)}{\Delta t} = \mathcal{L}[\overline{\phi(\mathbf{x},t)}^{Tr}], \qquad (2)$$

where $\hat{\mathbf{x}}$ denotes any Eulerian computational grid point, $\overline{(\ )}^{Tr}$ represents the average over a Lagrangian path connecting $(\hat{\mathbf{x}}, t+\Delta t)$ and $(\hat{\mathbf{x}} - \vec{V}\Delta t, t)$ in time-space domain. In the present model, an averaging based on an explicit/implicit weighting is used.

$$\overline{\phi(\mathbf{x},t)}^{Tr} = \alpha\phi(\hat{\mathbf{x}},t+\Delta t) + \beta\phi(\hat{\mathbf{x}} - \vec{V}\Delta t, t], \qquad (3)$$

where α and β are the explicit/implicit factors, and $\alpha + \beta = 1$. It is obvious that an $O(\Delta t^2)$ time integration can be obtained if $\alpha = \beta = 0.5$.

The numerical integrations of the governing equations are then written in form of (1). Let the updated value ϕ^{n+1} be at grid point $\hat{\mathbf{x}}$ at time $t+\Delta t$ andϕ^* the value at the departure point $\hat{\mathbf{x}} - \vec{V}\Delta t$ at time t which is re-mapped onto grid point $\hat{\mathbf{x}}$ by the CIP method[2,3], a Helmholtz equation for pressure can be derived from the time-discretized form of the governing equations. A multigrid algorithm is used to solve the pressure equation,and velocity and temperature are then computed with the updated pressure.

As the preliminary tests for the numerical model, we present two simulations as follow.

Density current: A density current was produced by releasing a cool dam[4]. Again, the density front and the eddies induced from Kelven-Helmholtz instability are well resolved even with a relatively coarse computational mesh (Fig.1).

Figure 1: Density current with a grid spacing of 250m × 250m.

Linear hydrostatic mountain wave: A linear mountain wave with small amplitude defined as the A1 test of Satomura's benchmark set[5] was simulated. Fig.2 shows the vertical velocity. The numerical solution recovered the vertical wave length and wave chain pattern with an adequate accuracy.

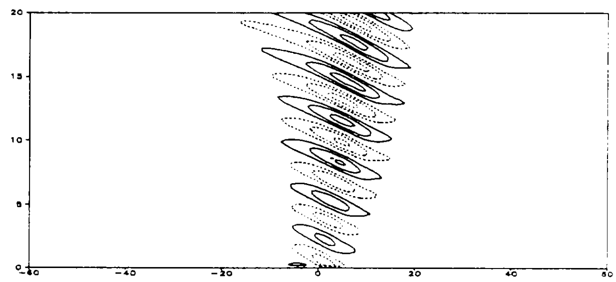

Figure 2: Vertical velocity of a linear hydrostatic mountain wave.

References

[1] M.Tanguay et.al, *Mon. Wea. Rev.* **118**(1990) 1970. [2] T. Yabe et.al, *Comput. Phys. Commun.* **66**(1991) 233 [3] F.Xiao, *Mon. Wea. Rev.* **128**(2000) 1165. [4] R.C.Carpenter et al. *Mon. Wea. Rev.* **118**(1990) 587.[5] T.Satomura, *http://www.clim.kugi.kyoto-u.ac.jp/satomura/*.