Modeling of Phenomena inside an Oscillating Bubble

Boonchai LERTNUWAT, Dept. of Mechanical Engineering, The University of Tokyo, Tokyo
Kazuyasu SUGIYAMA, Ship Performance Division, National Maritime Research Institute, Tokyo
Yoichiro MATSUMOTO, Dept. of Mechanical Engineering, The University of Tokyo, Tokyo

In the present study, a model of the thermal behavior inside an oscillating spherical bubble is developed coupling with the Rayleigh-Plesset equation. The present model is obtained by assuming that pressure gradient inside an oscillating bubble is uniform and the temperature gradient at bubble wall is linear. With the present model, thermal damping effect will be added into bubble behavior. This makes simulation close to the real phenomenon. Finally the behaviors of a bubble in an acoustic field are obtained by the full DNS, the present model and some other models. The results of several methods are compared and discussed. And it is found that the present model gives result agreeing well with DNS result but it requires less memory resource and computational time.

Introduction

Cavitation is understood as the rapid growth and collapse processes of the cavitation nuclei in water, which consist of unsolvable small gas bubbles, with evaporation and condensation through the bubble surface according to the surrounding pressure change. Naturally water always contains some gas impurities as small bubble nuclei. In the case that there is highly change of pressure or in the case that flow pressure reduces close to saturate vapor pressure of the liquid, bubble nucleus will show up their effects. Therefore it is also necessary to consider bubble effect to the whole flow in these cases. And the dynamic behavior of volumetric oscillating bubbles is important in the cavitating flow. The bubble oscillation is controlled by three damping factors: acoustical, viscous and thermal effects(1). The acoustical and viscous effects are explicitly contained in the equation of a bubble wall motion, e.g. the Rayleigh-Plesset or Keller equation(3), while the thermal effect is not. Most of former research works calculated the bubble oscillation under the assumption that gases inside a bubble changed isothermally or adiabatically, etc. Thermal effect was neglected in those cases. This is fine in some cases but not all. According to Chapman and Plesset (1971)(2), the thermal effect is the most important for bubbles with size of 10^{-2}-10^{-1}mm. If carrying out the Direct Numerical Simulation (DNS), where mass, momentum and energy transports inside the bubble are directly solved, we can accurately predict the bubble oscillation taking the thermal damping effect into account. However, a lot of numerical resources are needed in the DNS. Therefore, making a simple model for the thermal effect has been studied for a long time. Chapman and Plesset (1971)(2) and Brennen (1995)(1) presented an effective viscosity model, where the acoustical and thermal damping effects were treated as an increase of the viscosity in the Rayleigh-Plesset equation. Prosperetti (1991)(5) studied the polytropic model to calculate the proper polytropic index used to describe the thermal behavior inside the bubble. Yongliang and Stephen (1995)(8) proposed an empirical model of pressure difference to the change of the cavitation bubble size. Matsumoto *et al*. (1998)(4) presented a simple thermal damping model for the numerical simulation of the cavitating flow around a hydrofoil under the assumption that the bubble behaves as if isothermally when expanding and adiabatically when shrinking. In this work, authors estimated the heat conduction at the bubble wall for a single bubble by a linear equation and the thermal damping effect was modeled using the gas diffusivity and the characteristic time of the oscillating bubbles. The results of the present model show that it agrees well with the result of DNS method.

Nomenclature

A	Tube sectional area
D	Gas diffusivity
K	Gas heat conductivity
P	Pressures
R	Bubble radius
S	Liquid tension
T	Temperature
f	Volume fraction
u	Velocity
γ	Ratio of specific heat
μ	Liquid viscosity
ρ	Density
$\Re$	Gas constant
subscripts and superscripts	
l	Liquid
g	Gas
v	Vapor
b	Bubble

Modeling method

According to Prosperetti(5), under the assumption of the uniform pressure distribution inside the bubble, the following relation is obtained.

$$\frac{d}{dt}\left(P_g R^{3\gamma}\right) = 3(\gamma - 1)R^{3\gamma-1}K\left.\frac{\partial T}{\partial r}\right|_{r=R} \qquad (1)$$

By assuming that the temperature gradient at bubble wall is linear, the temperature gradient at the bubble wall is scaled by the thickness of the temperature boundary layer using gas diffusivity (D) and the characteristic time of the bubble (t_0) and expressed as,

$$\left.\frac{\partial T}{\partial r}\right|_{r=R} = \frac{(T_b - T_s)}{(Dt_0)^{1/2}}, \tag{2}$$

where $t_0 = 1/\omega_0$, ω_0 is the natural frequency of the bubble[1] and expressed as

$$\omega_0 = \left(\frac{3\gamma(P_l - P_V)}{\rho_L R_0^2} + \frac{2(3\gamma - 1)S}{\rho_L R_0^3} - \frac{4\upsilon_L^2}{R_0^4}\right)^{\frac{1}{2}}. \tag{3}$$

Moreover the vaporization on the bubble surface is considered using the following equation[6].

$$\dot{M}_v = \frac{0.4(P_s - P_v)}{\sqrt{2\pi\Re T_l}}, \tag{4}$$

where $\dot{M}_v$ is the vapor mass transfer rate at the bubble wall, P_s is the saturated pressure of the liquid phase and T_l is temperature of liquid.

Testing the present model

To ensure weather or not the present model works well, the present model is used to simulate flow inside a venturi. The venturi is as shown below.

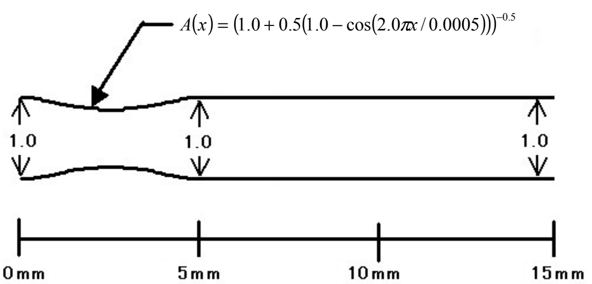

Fig. 1 Schematic of venturi tube

At the inlet, flow velocity, pressure, bubble radius and void fraction are set to be 10m/s, 50kPa, 0.1mm, respectively. The bubble size is chosen because it is the size that thermal damping effect is dominant[2].

To simulate flow in venturi, it requires some more governing equations.

- Conservation Equation of Liquid Volume Fraction:

$$\frac{\partial f_l}{\partial t} + \frac{1}{A}\frac{\partial f_l u_l A}{\partial x} = 0 \tag{5}$$

- Momentum Conservation Equation:

$$\frac{\partial f_l u_l}{\partial t} + \frac{1}{A}\frac{\partial f_l u_l^2 A}{\partial x} = -\frac{1}{\rho_l}\frac{\partial P}{\partial x}, \tag{6}$$

where $P = f_l P_l + f_g(P_g + P_v)$.

- Conservation Equation of Number Density of Bubbles:

$$\frac{\partial n_g}{\partial t} + \frac{1}{A}\frac{\partial n_g u_g A}{\partial x} = 0 \tag{7}$$

- Volume Fraction Relation:

$$f_l + f_g = 1, \tag{8}$$

where $f_g = 4/3\ \pi R^3 n_g$.

- Equation of a Bubble Translational Motion:

$$\frac{D_g}{Dt}\left(\beta\rho_l \frac{4}{3}\pi R^3 u_g\right) - \frac{D_l}{Dt}\left(\beta\rho_l \frac{4}{3}\pi R^3 u_l\right) = -F_P - F_D, \tag{9}$$

where,

$$\frac{D_g}{Dt} = \frac{\partial}{\partial t} + u_g\frac{\partial}{\partial x},$$

$$\frac{D_l}{Dt} = \frac{\partial}{\partial t} + u_l\frac{\partial}{\partial x},$$

$$F_P = -\frac{4}{3}\pi R^3\left(\nabla P_l + \mu\nabla^2 u_l\right),$$

$$F_D = \frac{1}{2}\pi R^2 \rho_l C_D |u_g - u_l|(u_g - u_l),$$

$$C_D = \frac{24}{\mathrm{Re}_{bub}}\left(1 + 0.15\mathrm{Re}_{bub}^{0.687}\right),$$

$$\mathrm{Re}_{bub} = \frac{2R\rho_l|u_g - u_l|}{\mu_l}.$$

- Equation of a Bubble Volumetric Motion (Rayleigh-Plesset Equation):

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{P_g + P_v - P_l}{\rho_l} - \frac{2S}{\rho_l R} - \frac{4\mu\dot{R}}{\rho_l R} \tag{10}$$

Flow will pass through the tube from left to right. Bubble will be activated and start to grow up just before the throat of venturi. Then it oscillates in the down stream part. And in this down stream part of the venturi, the damping effect will show up itself. In the case of strong damping effect, bubble oscillation will be retarded quickly and reach its steady state. In contrast, bubble oscillation changes just little in the case of weak damping effect.

The result from the present model will be compared with result of other models to see whether or not it is acceptable. The other models employed here are isothermal model and adiabatic model. Besides DNS[7] result is also shown for comparing.

In the case of isothermal model, pressure inside bubble is expressed by Eq. (11).

$$P_g R^3 = const. \tag{11}$$

In the case of adiabatic model, pressure inside bubble is expressed by Eq. (12).

$$P_g R^{3\gamma} = const. \tag{12}$$

While DNS result is obtained from the algorithm presented in papers of Takemura *et al*. (1994)[7]. Mass, momentum and energy transports inside the bubble are directly solved. So we can accurately predict the bubble oscillation taking the thermal damping effect into account.

Results

At first, bubble profile is shown in Fig. 2. This figure shows how bubble changes its size along the flow from left to right. Bubble starts to grow up just before the throat of the venturi. Then it reaches the maximum size before starting to oscillate in the down stream portion.

Result from four different computational methods is compared. And it is found that result from isothermal model is the most different from the others. At the first loop of oscillation, DNS, present model and adiabatic model give almost the same result. But adiabatic model shows up different result just beyond the first collapsing process. In the case of adiabatic model, the maximum bubble radius size of the second, the third and the forth loop are almost the same. While in the case of DNS and the present model, they reduce along down stream flow. However, there is still some difference between the result of DNS and the present model. But much less than the other two cases.

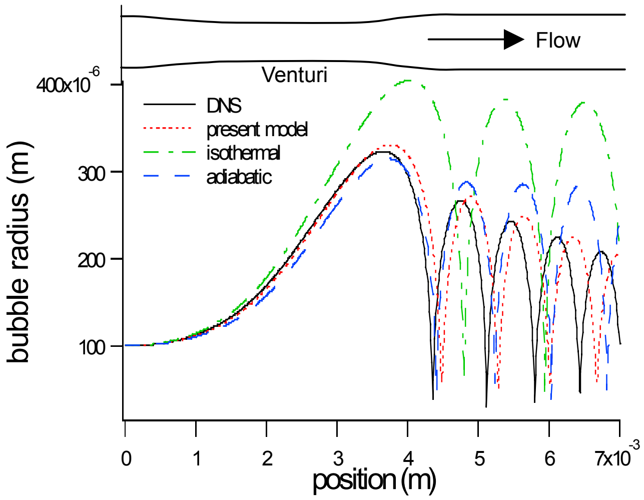

Fig. 2 Comparison of bubble radius distributions in tested venturi amount different simulation methods.

The profile of gas pressure inside bubble is also considered. The result of four simulation methods is shown in Fig. 3.

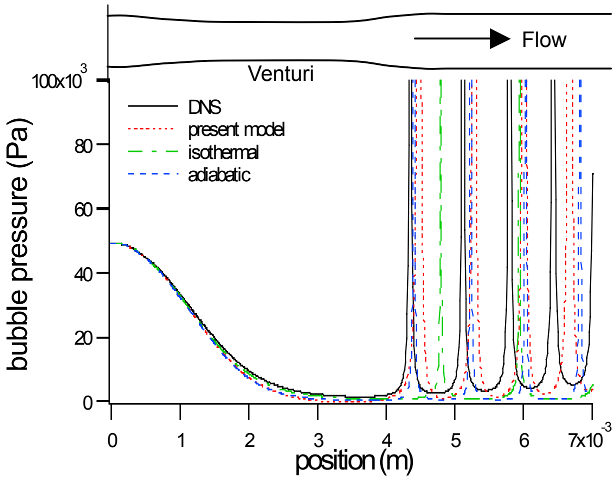

Fig. 3 Comparison of gas pressure inside bubble distributions in tested venturi amount different simulation methods.

Gas pressure inside bubble reduces from the initial value to nearly zero (at position between 2~4x10^{-3}m). This can tell us that cavitation has occurred. And it is clear that there are some difference amount results at the cavitation of the first oscillation loop. When cavitation occurs, the present model gives the lowest cavitating pressure while DNS gives the highest cavitating pressure. But all is in the same order anyway. For the second loop, isothermal and adiabatic model give the lowest pressure in the same order of the first loop. Whereas DNS and the present model give the lowest pressure in the second loop different from the first loop. This is the same for the case of the third loop, the forth loop and so forth.

Since the present model is derived from temperature gradient at bubble wall as shown in Eq. (2), it is to verify this value also. Figure 4 shows the temperature gradient at bubble wall computed by DNS. And Figure 5 shows the result of the present model. From both figures, result of DNS case and the present model case are almost the same. The difference between results of both methods is at collapsing process. For the case of DNS, after bubble reaches the smallest size, temperature gradient overshoots to a certain value. But it is different in the case of present model. Temperature gradient at bubble wall computed by the present model smoothly changes after bubble rebounds from the smallest size.

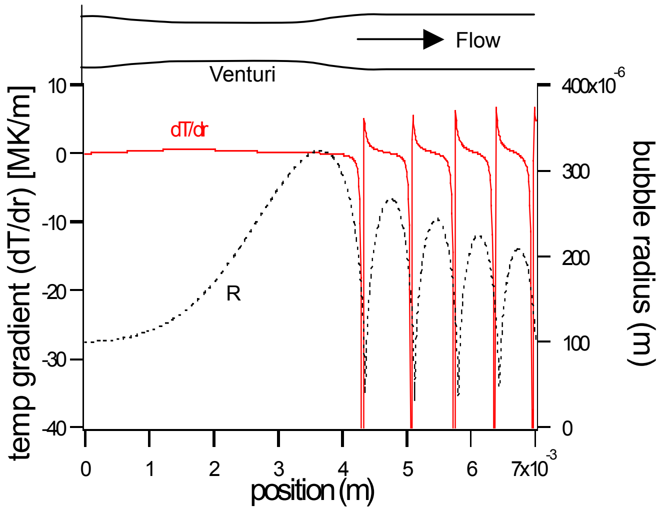

Fig. 4 Profile of temperature gradient at bubble wall and bubble radius in venturi tube computed by DNS.

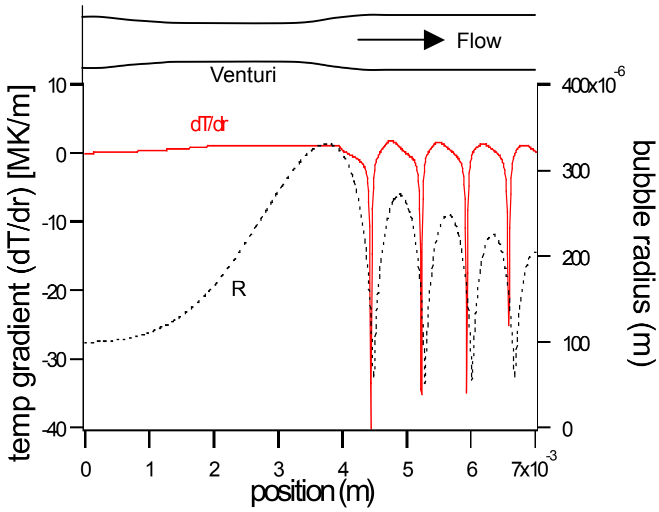

Fig. 5 Profile of temperature gradient at bubble wall and bubble radius in venturi tube computed by the present model.

Discussion

According to figure 2, isothermal bubble expands larger than others. This is the result from fixing temperature to be constant. When temperature inside bubble is fixed, gas pressure inside bubble relates to bubble radius by Eq. (11). That is gas pressure varies with R^{-3}. But in the case of adiabatic model, gas pressure varies with $R^{-3\gamma}$. Therefore in the case of isothermal model, gas pressure varies less for the same radius change. This is the reason why isothermal bubble grows larger than adiabatic bubble. Let us consider Eq. (10) (Rayleigh-Plesset Equation). The first term of the right hand side refers to pressure difference between outside and inside bubble. At venturi throat, flow velocity is very high. Liquid pressure (P_l), thus, reduces. This makes bubble to grow up with positive $\ddot{R}$. If bubble grows up gas pressure inside bubble reduces. When gas pressure reduces and reaches a certain value, the first term of the right had side of Eq. (10) will be negative. And bubble starts to collapses with minus $\ddot{R}$. Consequently, isothermal bubble requires more change of bubble radius before gas pressure reaches the minimum value, because gas pressure changes slowly with bubble radius. So it is important to select the proper thermodynamics process to govern the phenomena inside a bubble to get the correct size of cavitation bubble.

DNS and the present model govern the thermal phenomena inside bubble differently. They calculate how much heat flow penetrates in to or leaks out off the oscillating bubble. Theoretically, the thermodynamics process of gas inside bubble should vary between isothermal process and adiabatic process. Although the computational condition used in this paper makes adiabatic bubble oscillates nearly the same as DNS and the present model case. But it may be different for other computational conditions.

After the first collapsing process, it is clear that bubble is damped in the case of DNS and the present model. But in the case of isothermal and adiabatic model, it looks like no damping effect, namely bubble returns to the same size before it collapses. Besides the frequency of bubble oscillation is different.

Figure 2 shows only the absolute value of results. But in this paper, relative value of results is considered as well, so that we can see if or not the results from the present model agree well with DNS. The interested relative values are maximum bubble radius at each oscillation loop (R_{max}) and the position where they occur (X_{max}). (See Figure 6 for detail.)

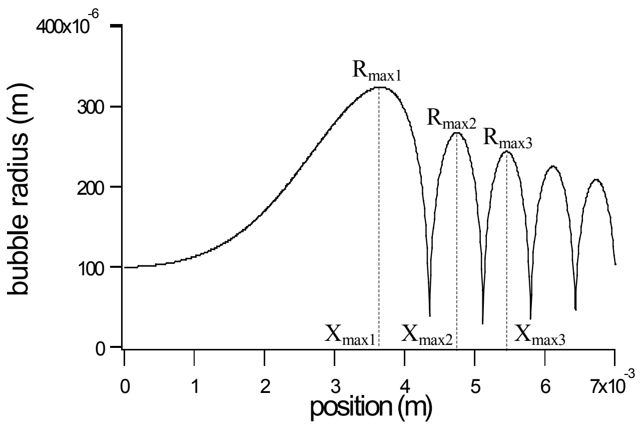

Fig. 6 Schematic for showing the values that are taken to calculate relatively.

By using R_{max1} as a reference, the relative value of $R_{max(i)}/R_{max1}$ are plotted in Figure 7 (Here i is loop number = 1, 2, 3, …). And Figure 8 is obtained by the same way but it is for relative X_{max}.

Figure 7 shows that maximum bubble at each oscillation loop reduces from upstream to downstream. But it reduces slowly in isothermal and adiabatic case whereas it reduces quickly in the case of DNS and present model. Figure 5 can explain the reason why results are different. Accordance with figure 5, the temperature gradient at bubble wall is extremely negative at the point of collapsing. And from Eq. (1), we can see that negative temperature gradient at bubble wall makes gas pressure reduced at the next time step. Then we get lower $\ddot{R}$ from Eq. (10) because of the reduced gas pressure. Consequently, bubble rebounds with lower radius speed ($\dot{R}$). And it is the same in the case of DNS. Physically this means that bubble loses some energy at each collapsing process. This can be considered as either cavitation noise or cavitation luminescence. But they are not considered here because they are out of the scope of this work. Return to figure 7, it is found that maximum bubble radius reduces lower than 50% at the 10th loop if thermal damping effect is considered. But it reduces only 10% if thermal damping effect is neglected.

Figure 8 refers to the frequency of oscillation. The steeper graph slope is, the lower oscillation frequency is. Result of the present model agrees well with DNS. But isothermal model and adiabatic model give lower frequency. This is consistent to the size of bubble. In the case of isothermal model and adiabatic model, maximum rebounding bubble radius (of loop no. 2, 3, 4, …) is larger than case of DNS and the present model. So isothermal bubble and adiabatic bubble require more time to grow up and shrink down.

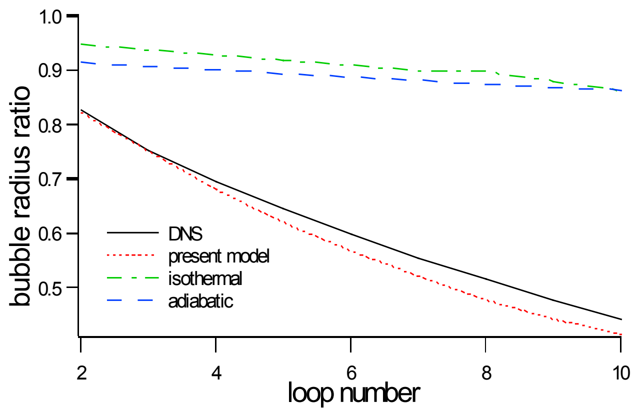

Fig. 7 Maximum bubble radius ($R_{max(i)}$) of each oscillation loop relative to the maximum bubble radius (R_{max1}) the first loop.

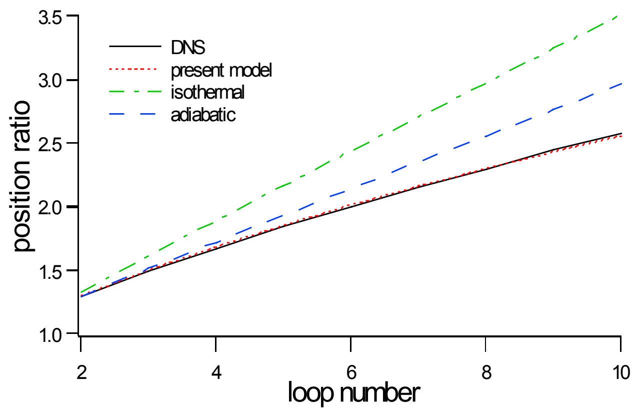

Fig. 8 The position of maximum bubble radius of each oscillation loop ($X_{max(i)}$) relative to the position of maximum bubble radius the first loop (X_{max1}).

Return to figure 3, gas pressure inside bubble is shown there. When cavitation occurs (at position between 2~4x10^{-3}m), it shows that gas pressure is the same for all cases. Gas pressure at cavitation of the present model case is less than that of DNS case. This agrees with result in figure 2. Because at position between 2~4x10^{-3}m bubble radius of the present model case is larger than that of DNS case. However the difference between the present model case and DNS case is so little if compared with isothermal case and adiabatic case.

Figure 4 and 5 show that there is some difference between temperature gradient of DNS case and the present model case. The difference between temperature gradient of these two cases results from non-linear effect. This is because Eq. (2) is modeled by assuming that the temperature gradient at bubble wall is linear. But as the matter of fact, the temperature gradient is almost linear for most of time except when bubble collapses. At this time, temperature gradient at bubble wall is no longer linear but it is non-linear. Although there is some difference between these two cases but it is not much enough to give unacceptable final result (bubble radius and pressure). Therefore it is acceptable to use the present model for governing thermal behavior inside an oscillating bubble.

Conclusions

1. Thermal behavior of gas content inside an oscillating bubble gives important thermal damping.
2. Conventional models (such as isothermal and adiabatic model) are lack of thermal damping results, so they are not proper to use to predict bubble behavior.
3. Present model shows quite good prediction of the bubble behavior. It requires much less computational time and memory than the DNS. Therefore, the present model will be useful for the numerical simulations of cavitating flows containing many bubbles

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